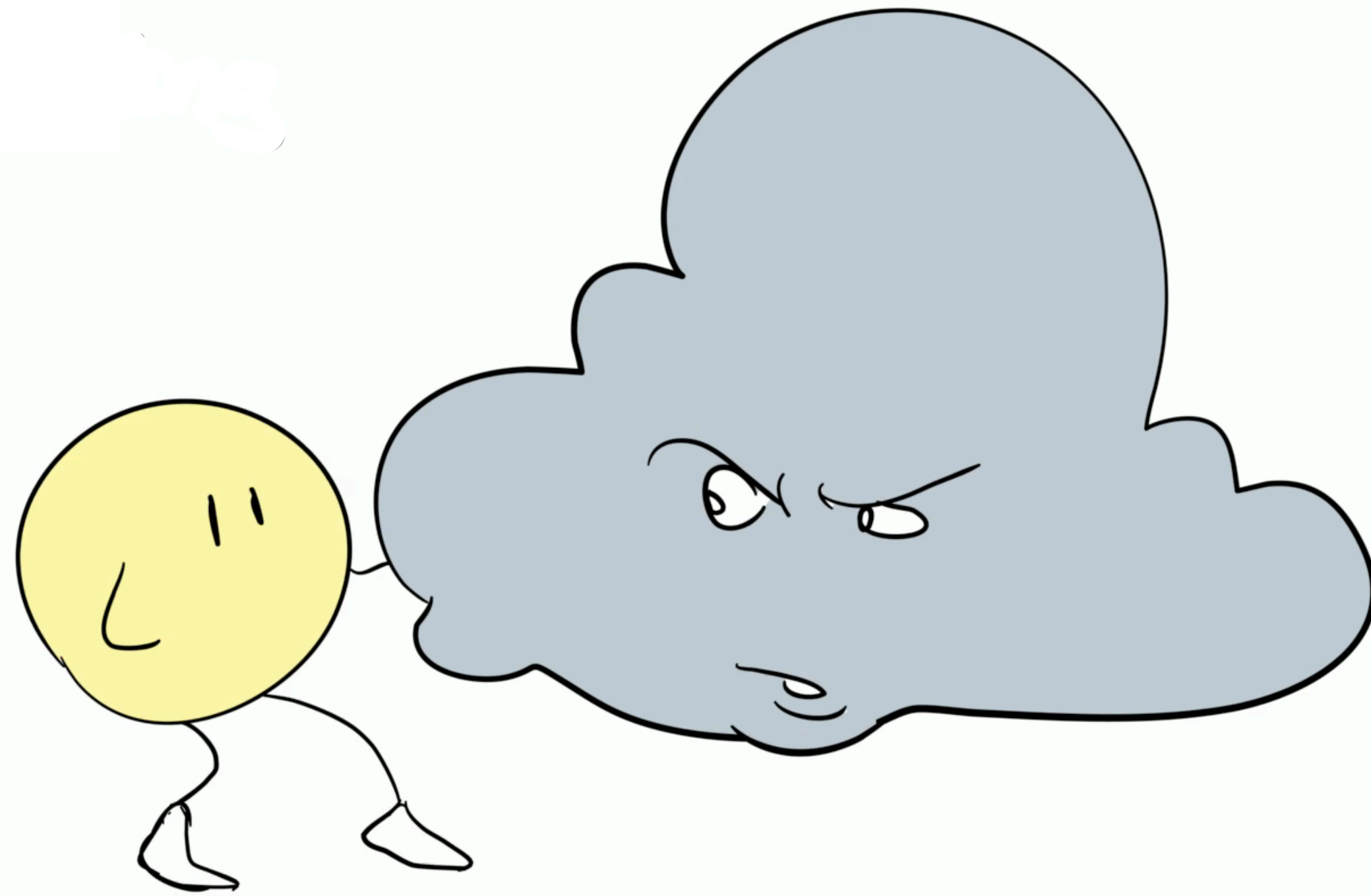


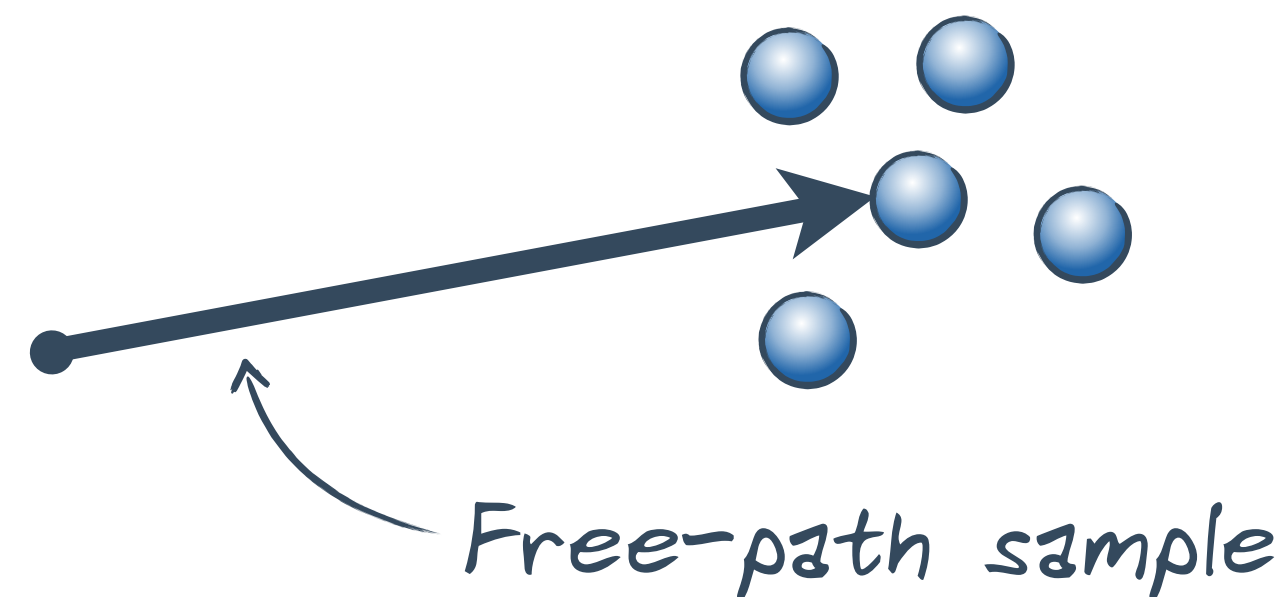
DISTANCE SAMPLING

Jan Novák
Disney Research



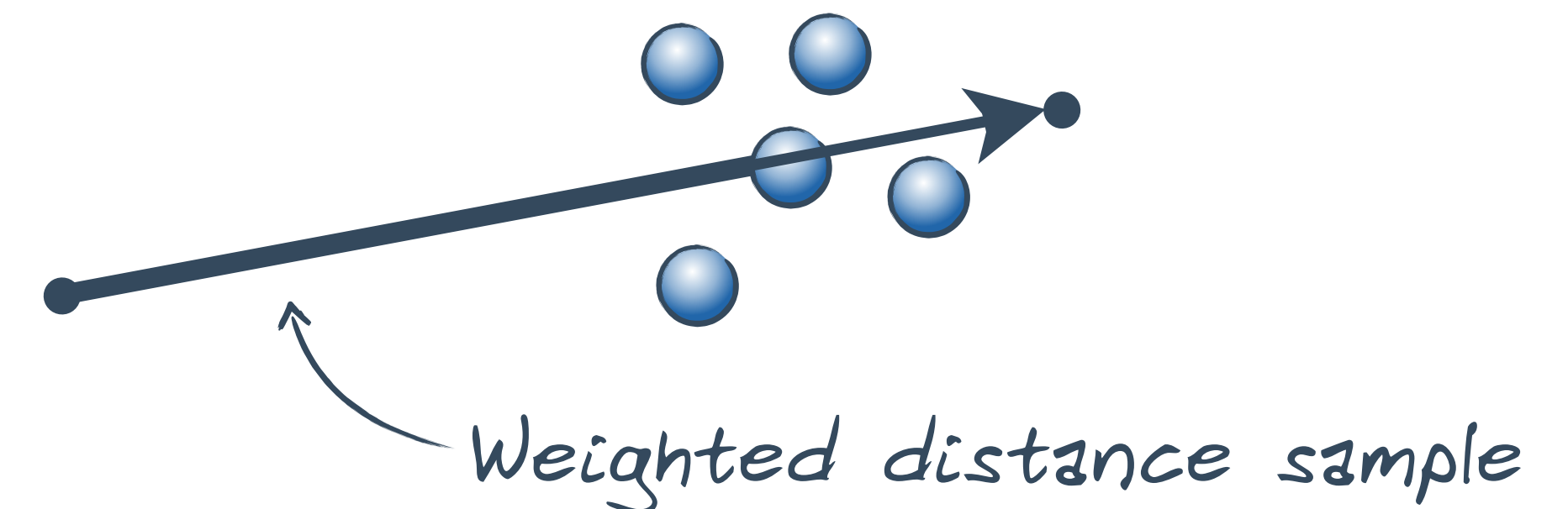
ANALOG methods

- ▶ Adhere to physical process
- ▶ Produce **free-path** samples
- ▶ Energy of particles **unchanged**



NON-ANALOG methods

- ▶ Deviate from physical process
- ▶ Produce **arbitrary distance** samples
- ▶ Particles (photons) are **weighted**



FREE-PATH SAMPLING

How to sample the flight distance to the next interaction?

$$T(t) = e^{-\int_0^t \mu_t(s) ds} = \begin{cases} P(X > t) \\ P(X \leq t) = F(t) \end{cases}$$

Random variable representing flight distance

Cumulative distr. function (CDF)

Partition of unity

$$F(t) = 1 - T(t)$$

Recipe for generating samples

FREE-PATH SAMPLING

Probability density function (**PDF**)

$$p(t) = \frac{dF(t)}{dt} = \frac{d}{dt} \left(1 - e^{-\tau(t)} \right) = \mu_t(t) e^{-\tau(t)}$$

Cumulative distribution function (**CDF**)

$$F(t) = 1 - T(t) = 1 - e^{-\tau(t)}$$

Inverted cumulative distr. function (**CDF⁻¹**)

$$\xi = 1 - e^{-\tau(t)} \quad \text{Solve for } t$$

$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

Approaches for finding t :

- 1) ANALYTIC (closed-form CDF⁻¹)
- 2) SEMI-ANALYTIC (regular tracking)
- 3) APPROXIMATE (ray marching)

Inverted cumulative distr. function (**CDF⁻¹**)

$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

Some simple volumes permit closed-form solutions

Example: **homogeneous** medium ($\mu_t(\mathbf{x}) = \mu_t$)

Opt. thickness

$$\int_0^t \mu_t(s) ds = t\mu_t$$

Inverted CDF

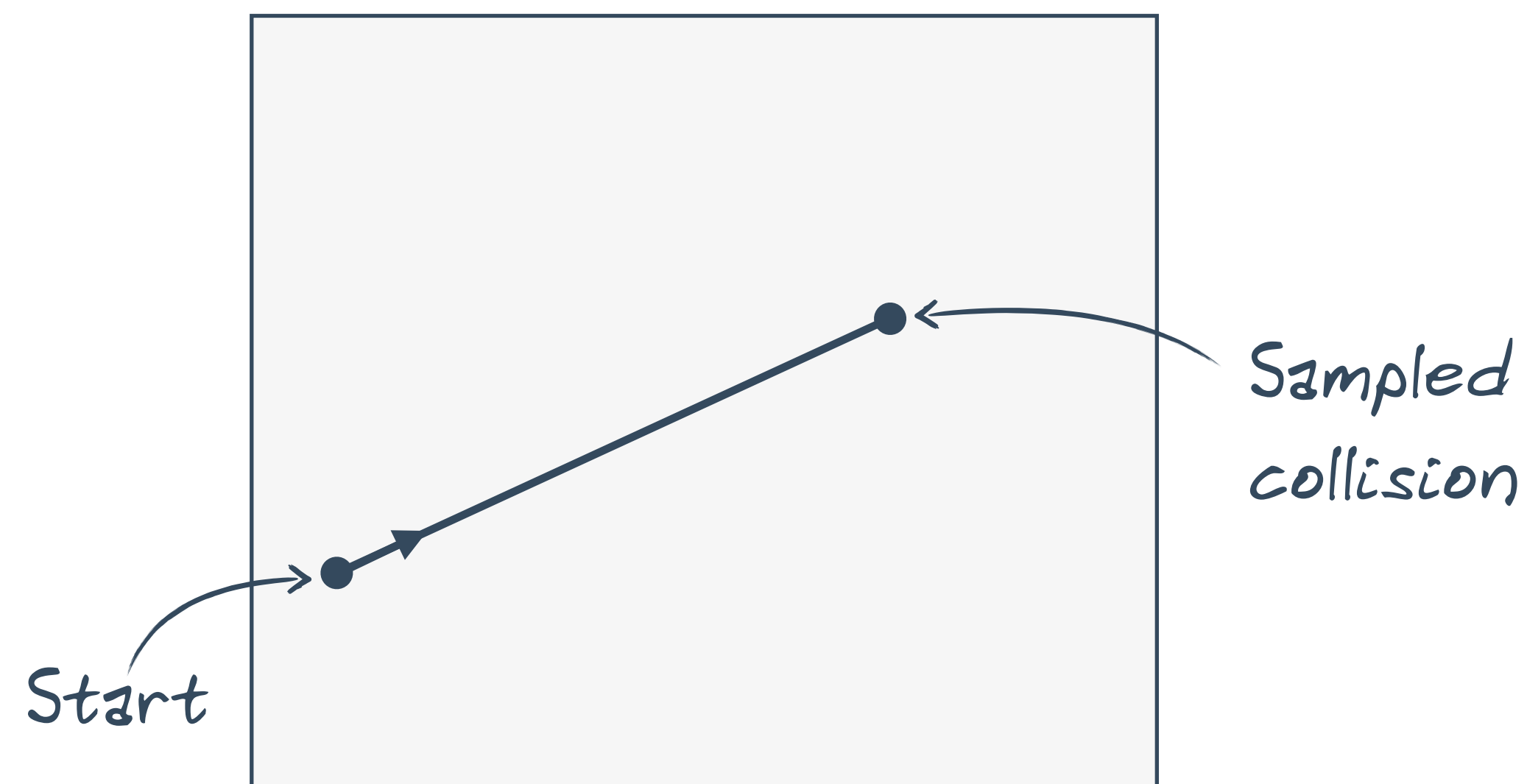
$$\Rightarrow F^{-1}(\xi) = -\frac{\ln(1 - \xi)}{\mu_t}$$

ANALYTIC APPROACH

Inverted cumulative distr. function (**CDF⁻¹**)

$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

Homogeneous volume



Sampling in homogeneous vol:

- 1) Draw a random number ξ
- 2) Set $t = -\frac{\ln(1 - \xi)}{\mu_t}$
- 3) Set $p(t) = \mu_t e^{-t\mu_t}$

REGULAR TRACKING (SEMI-ANALYTIC) EUROGRAPHICS 2018

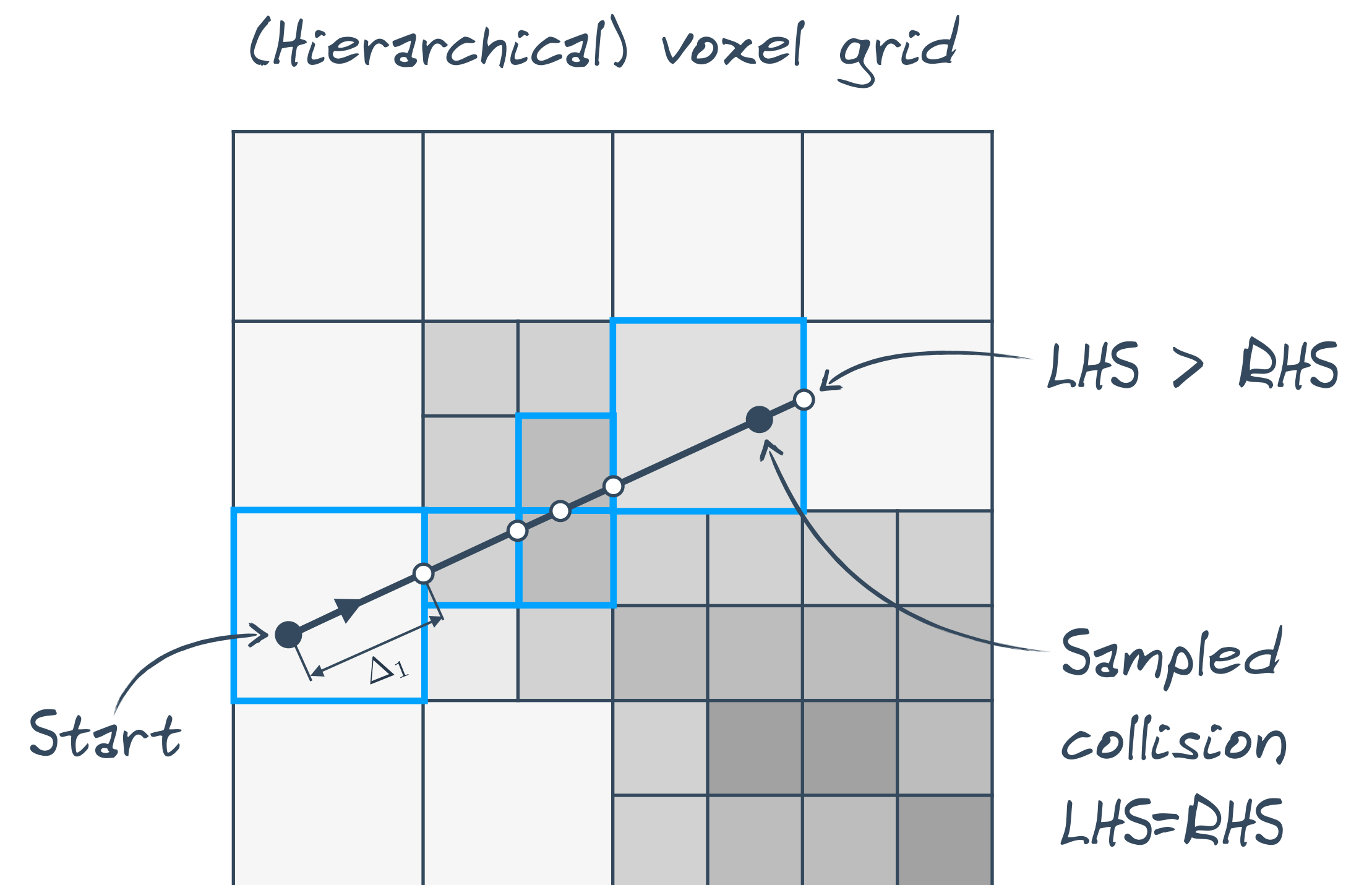
For piecewise-simple (e.g. piecewise-constant), summation replaces integration

$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

$$\sum_{i=1}^k \mu_{t,i} \Delta_i = -\ln(1 - \xi)$$

Regular tracking:

- 1) Draw a random number ξ
- 2) While LHS < RHS
move to the next intersection
- 3) Find the exact location
in the last segment analytically



RAY MARCHING

Find the collision distance approximately

$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

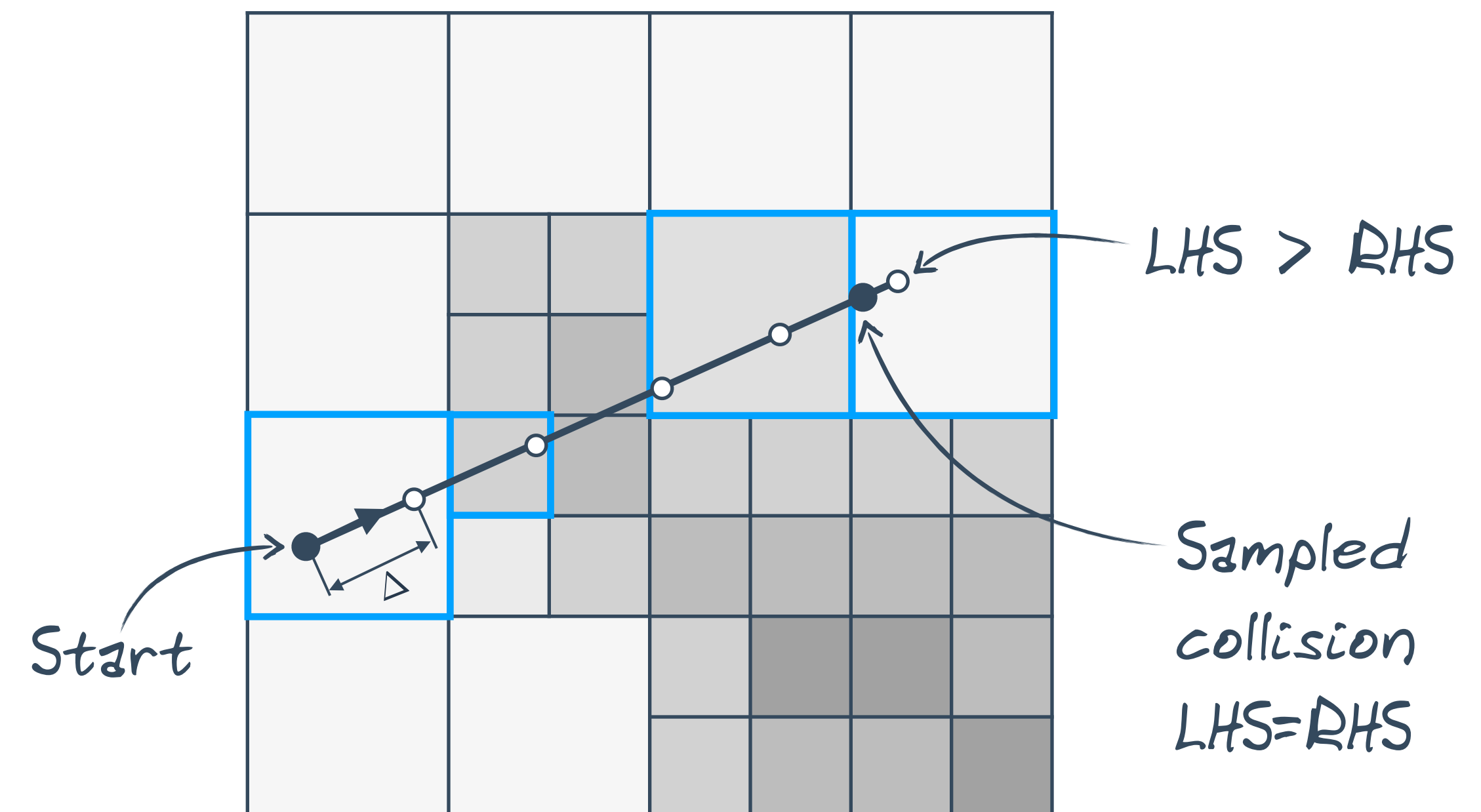
$$\sum_{i=1}^k \mu_{t,i} \Delta = -\ln(1 - \xi)$$

Constant step

Ray marching:

- 1) Draw a random number ξ
- 2) While LHS < RHS
make a (fixed-size) step
- 3) Find the exact location
in the last segment analytically

(Hierarchical) voxel grid



RAY MARCHING

Find the collision distance approximately

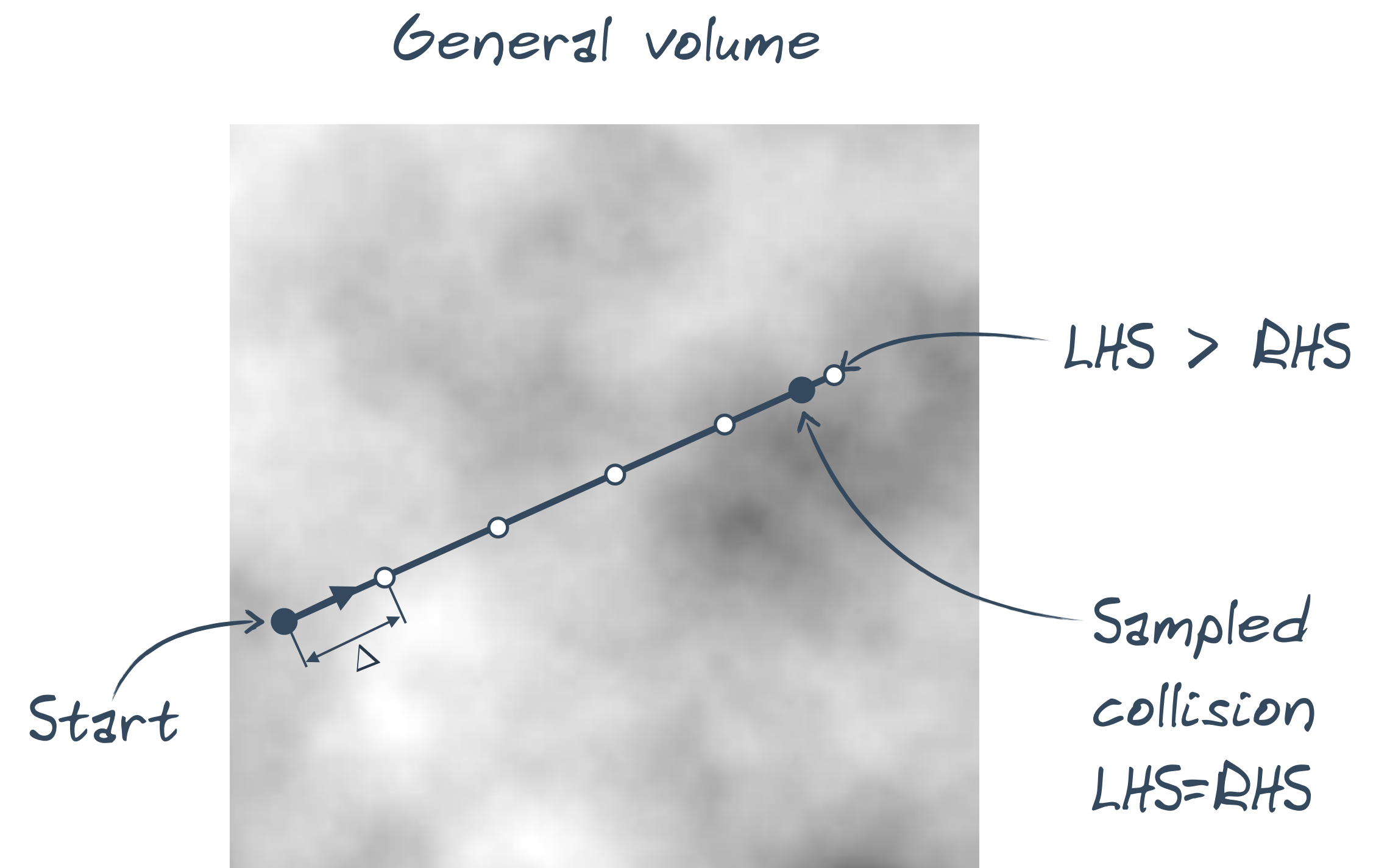
$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

$$\sum_{i=1}^k \mu_{t,i} \Delta = -\ln(1 - \xi)$$

Constant step

Ray marching:

- 1) Draw a random number ξ
- 2) While LHS < RHS
make a (fixed-size) step
- 3) Find the exact location
in the last segment analytically



RAY MARCHING

Find the collision distance approximately

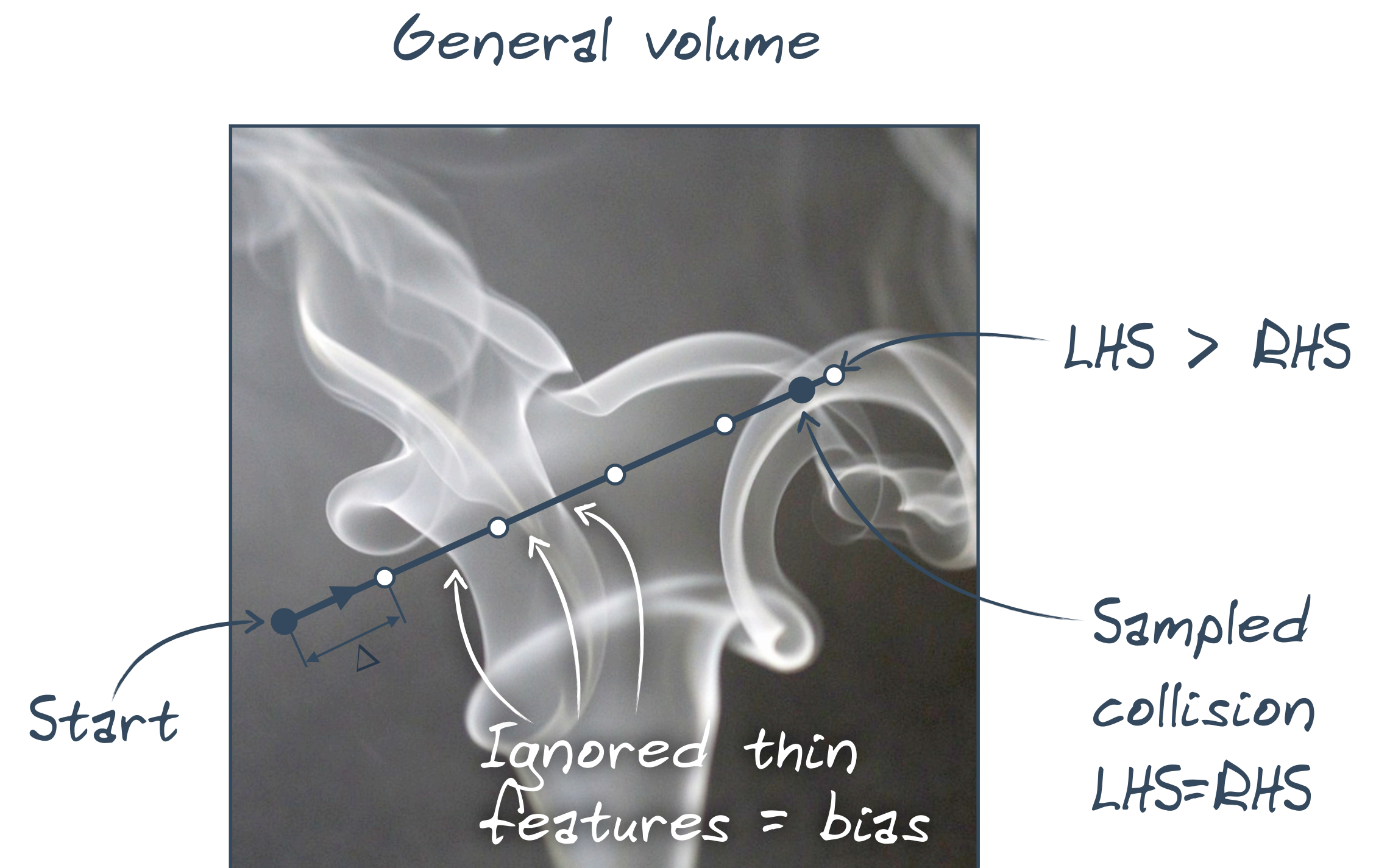
$$\int_0^t \mu_t(s) ds = -\ln(1 - \xi)$$

$$\sum_{i=1}^k \mu_{t,i} \Delta = -\ln(1 - \xi)$$

Constant step

Ray marching:

- 1) Draw a random number ξ
- 2) While LHS < RHS
make a (fixed-size) step
- 3) Find the exact location
in the last segment analytically



ANALYTIC CDF⁻¹

- ▶ Efficient & simple, limited to few volumes
- ▶ Simple volumes (e.g. homogeneous)
- ▶ Unbiased

REGULAR TRACKING

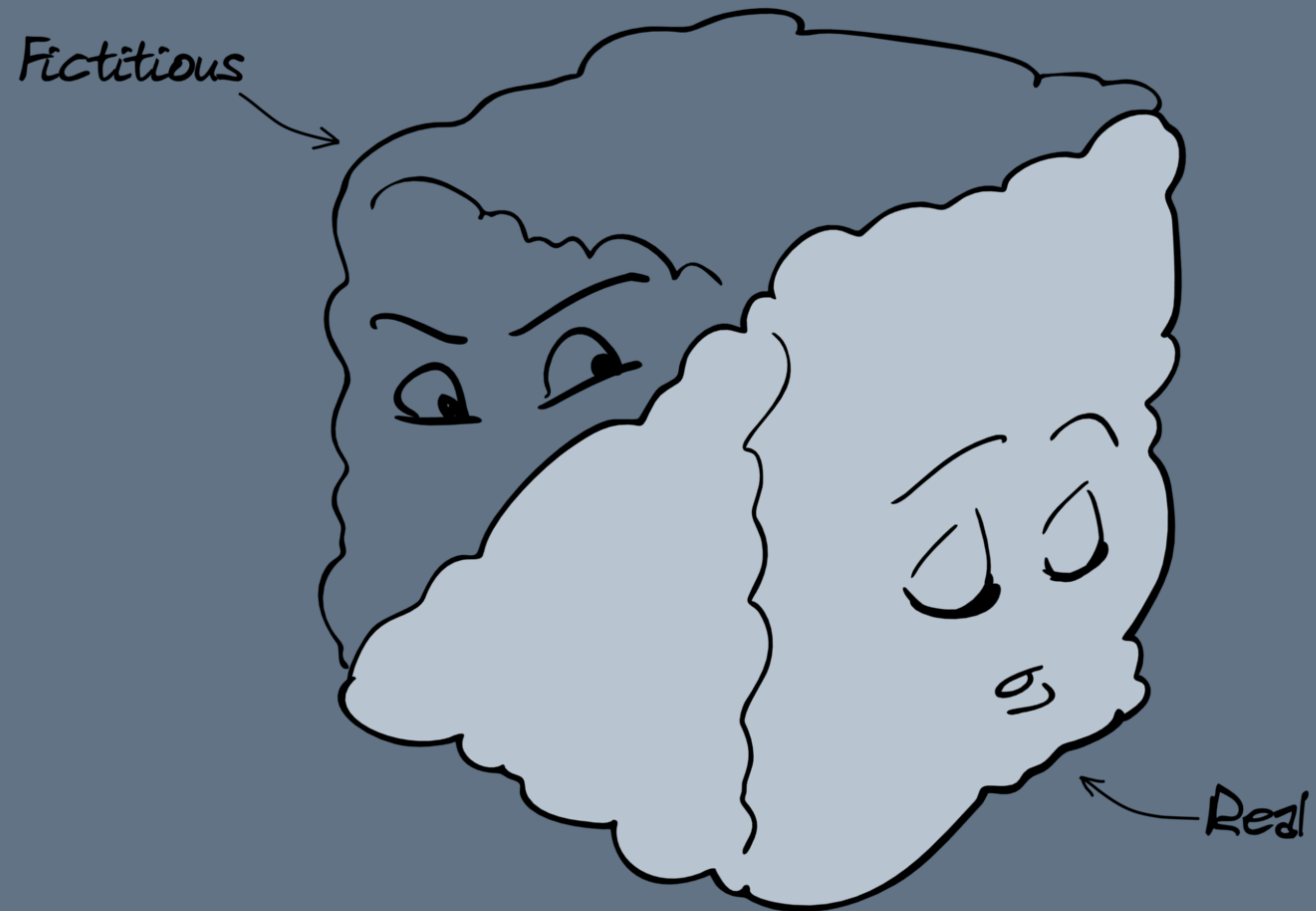
- ▶ Iterative, inefficient if free paths cross many boundaries
- ▶ Piecewise-simple volumes
- ▶ Unbiased

RAY MARCHING

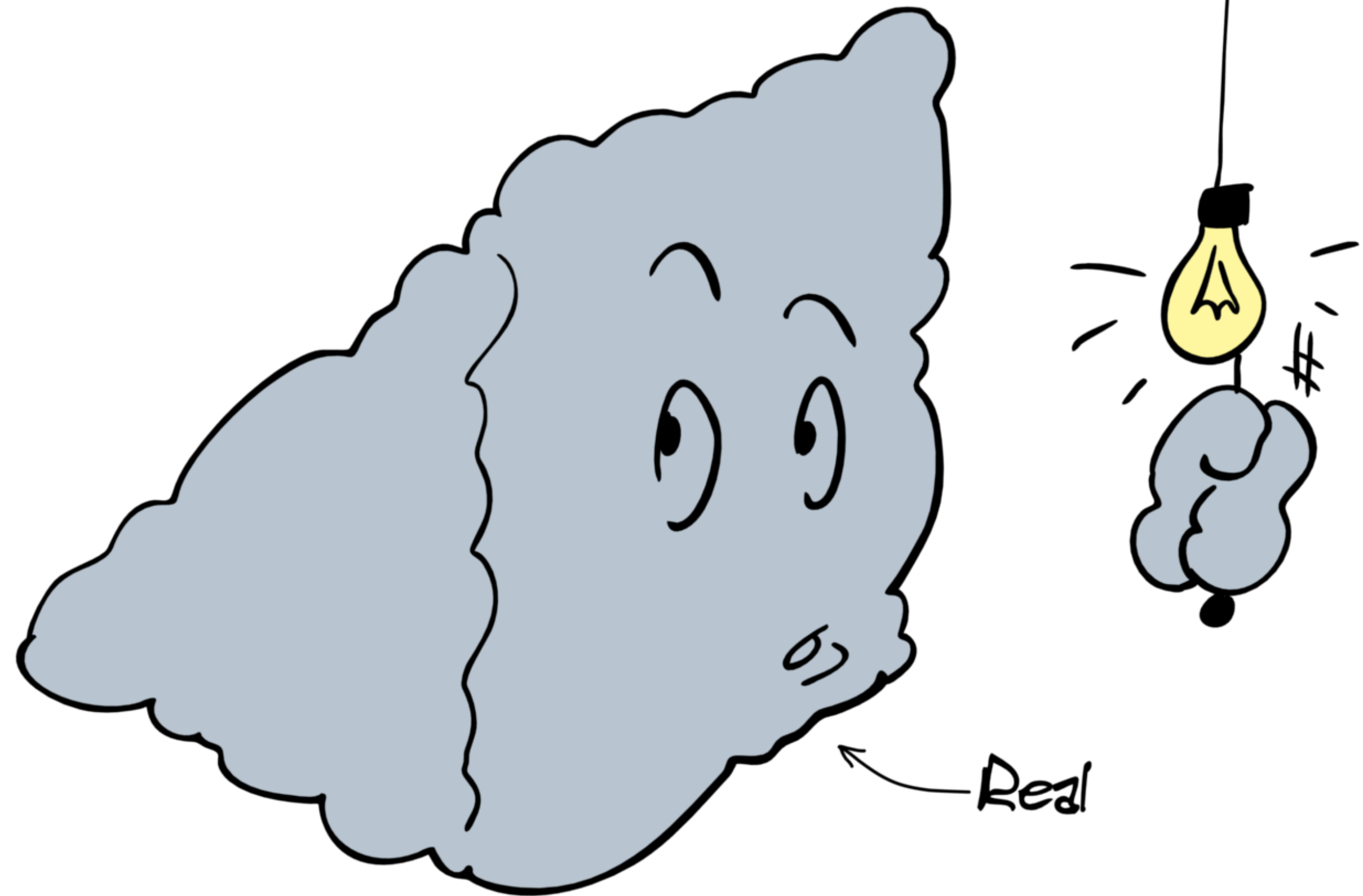
- ▶ Iterative, inaccurate (or inefficient) for media with high frequencies
- ▶ Any volume
- ▶ Biased

Common approach: sample optical thickness, find corresponding distance

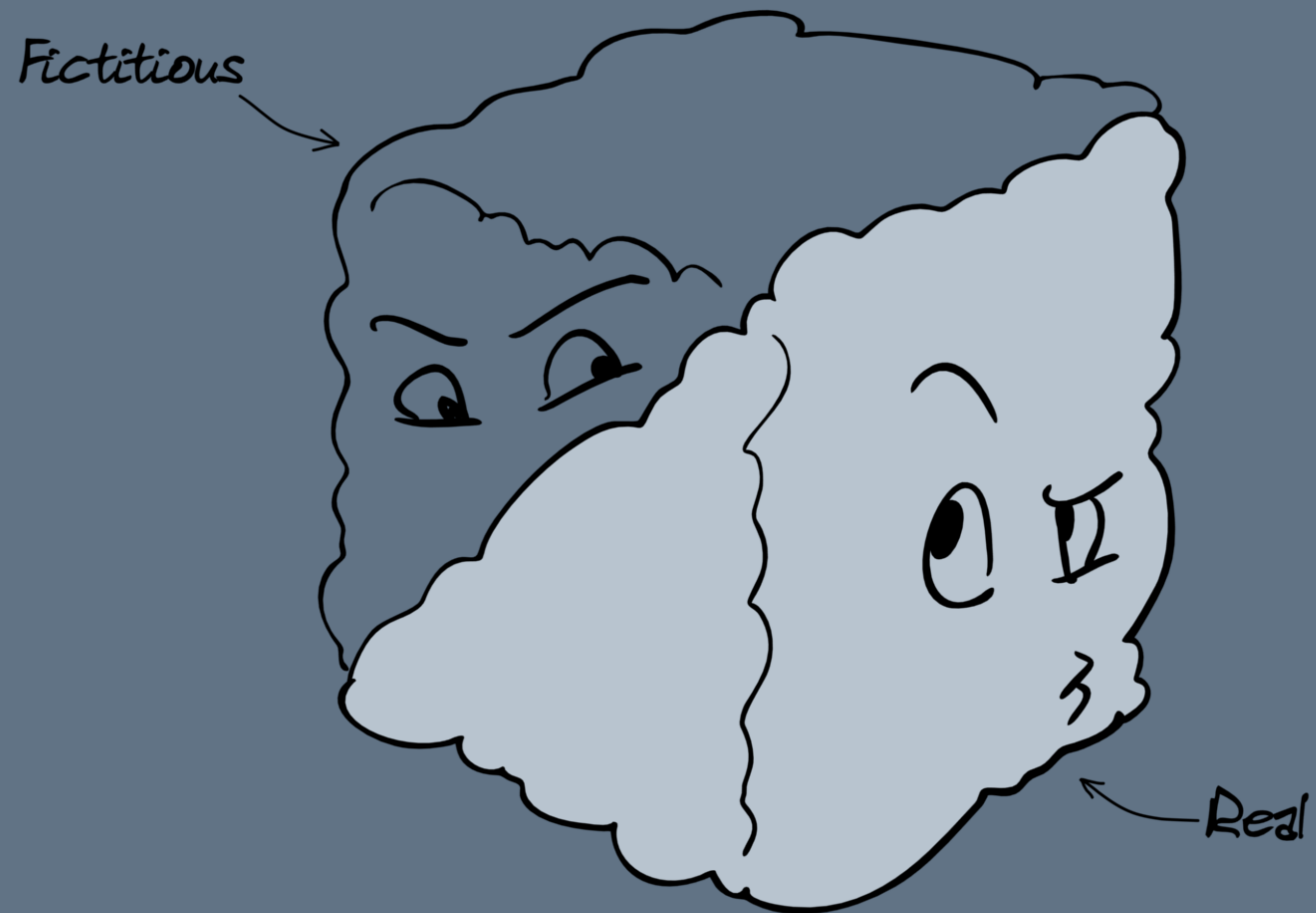
NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS



NULL-COLLISION ALGORITHMS

Origins in neutron transport and plasma physics, unbiased sampling

Applied in rendering since 2008 [Raab et al. 2008]

FREE-PATH sampling:

- ▶ **Delta tracking** (a.k.a Woodcock tracking)
- ▶ **Weighted delta tracking**
- ▶ **Decomposition tracking**
- ▶ **Spectral tracking**

*Discussed
by Jo later*

TRANSMITTANCE estimation:

- ▶ Delta tracking
- ▶ (Residual) ratio tracking
- ▶ Next-flight delta/ratio tracking

*Discussed together w/
other transmittance estimators*

DELTA TRACKING

WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

a.k.a. Woodcock tracking, pseudo scattering, hole tracking, null-collision method,...

PHYSICALLY-BASED interpretation

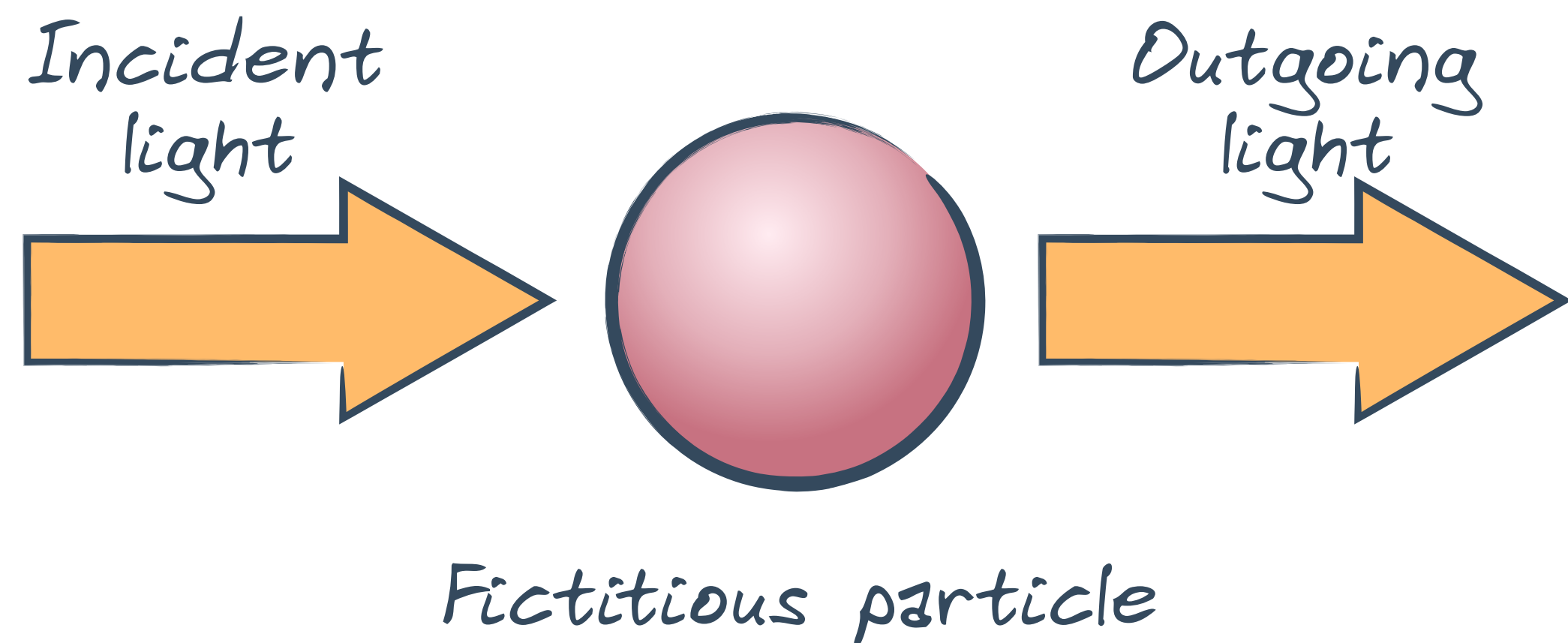
- ▶ Correctness motivated by intuitive physical arguments:
Butcher and Messel [1958, 1960],
Zerby et al. [1961], Bertini [1963],
Woodcock et al. [1965], Skullerud [1968],
...

MATHEMATICAL formalisms

- ▶ Proofs: Miller [1967], Coleman [1968]
- ▶ Integral formulation: Galtier et al. [2013]

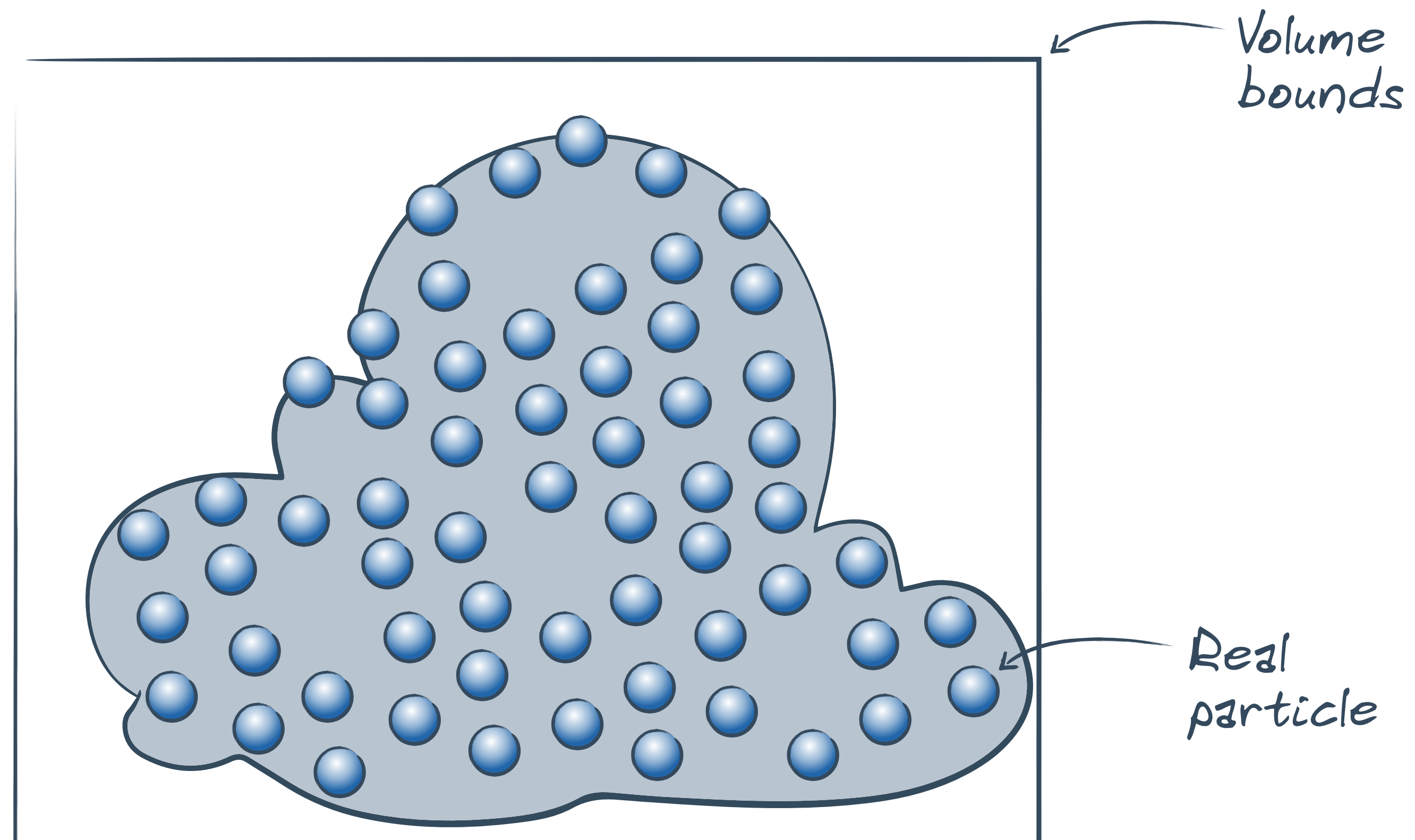
Add **FICTITIOUS MATTER** to homogenize heterogeneous extinction

- ▶ albedo $\alpha(\mathbf{x}) = 1$
- ▶ phase function $f_p(\omega, \bar{\omega}) = \delta(\omega - \bar{\omega})$

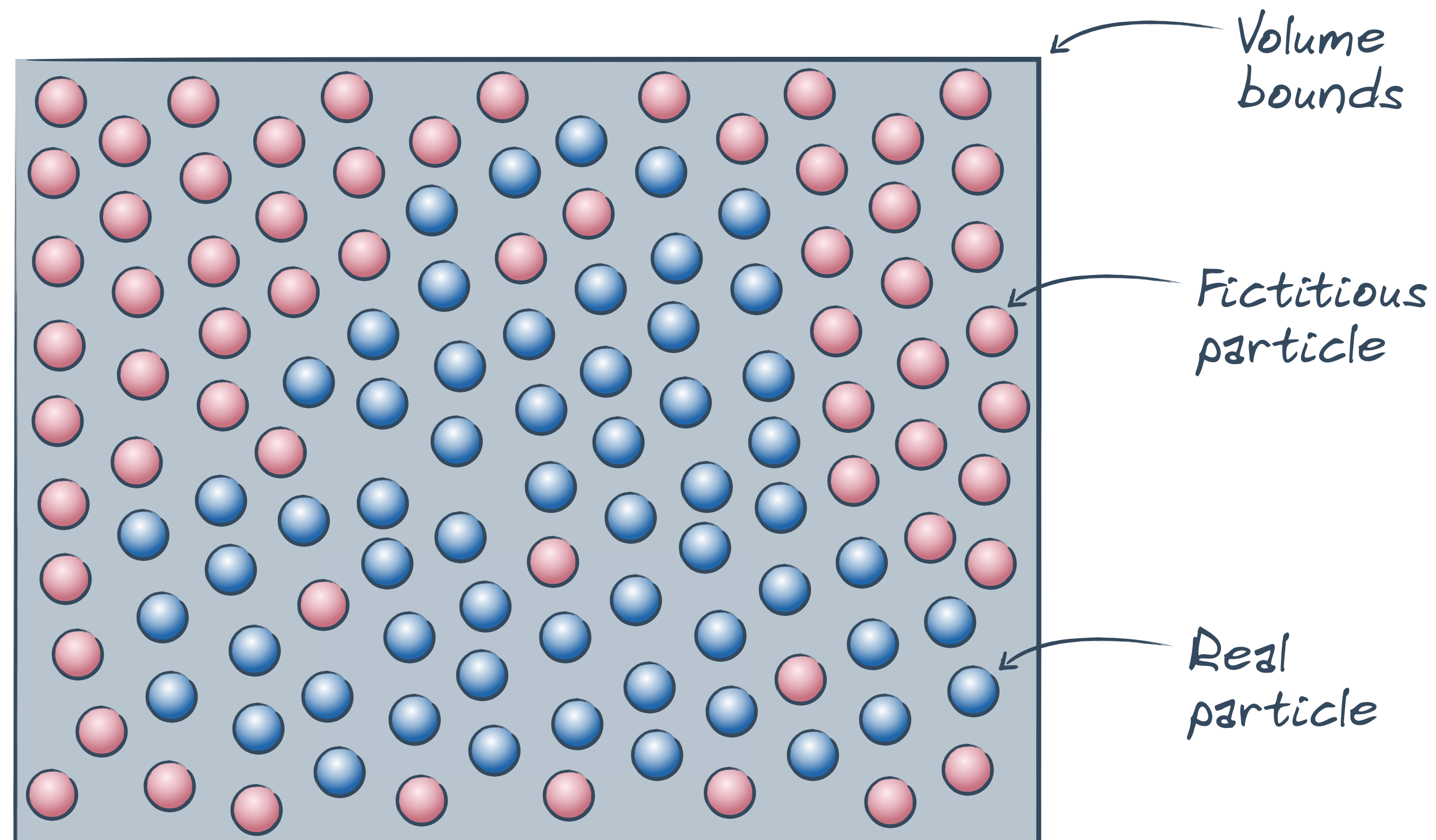


Presence of fictitious matter does not impact light transport

HOMOGENIZATION

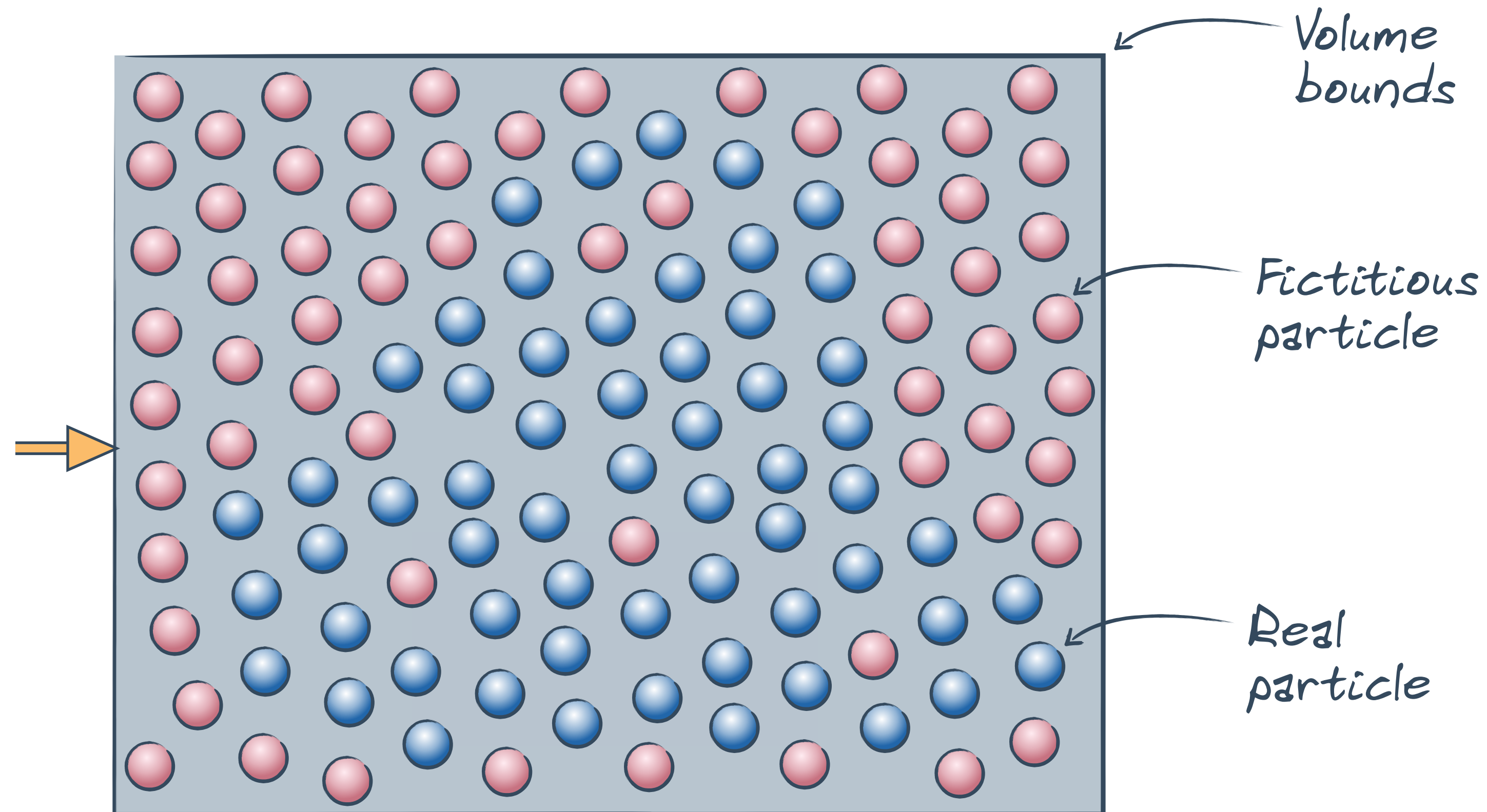


HOMOGENIZATION

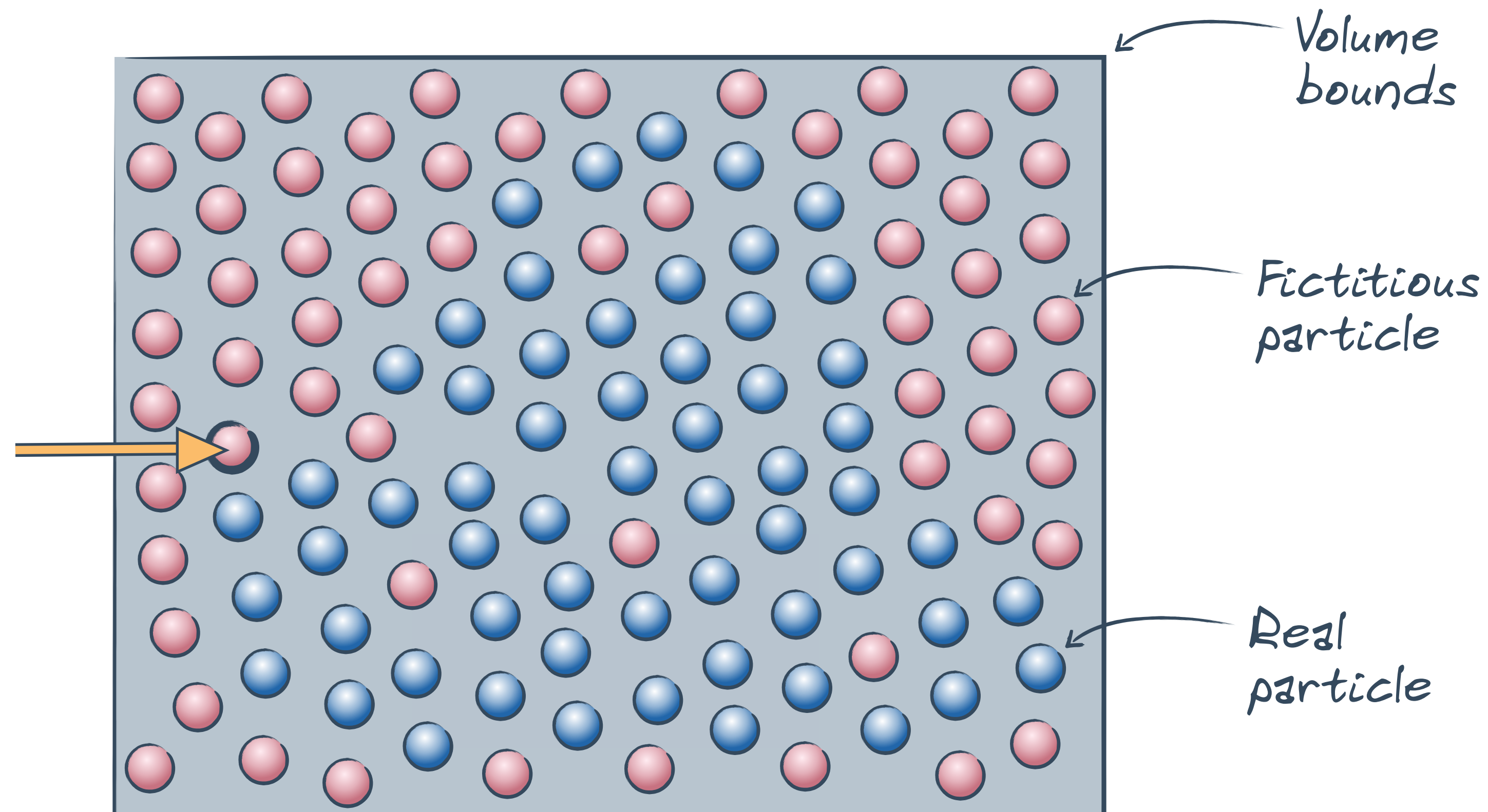


PHYSICAL INTERPRETATION

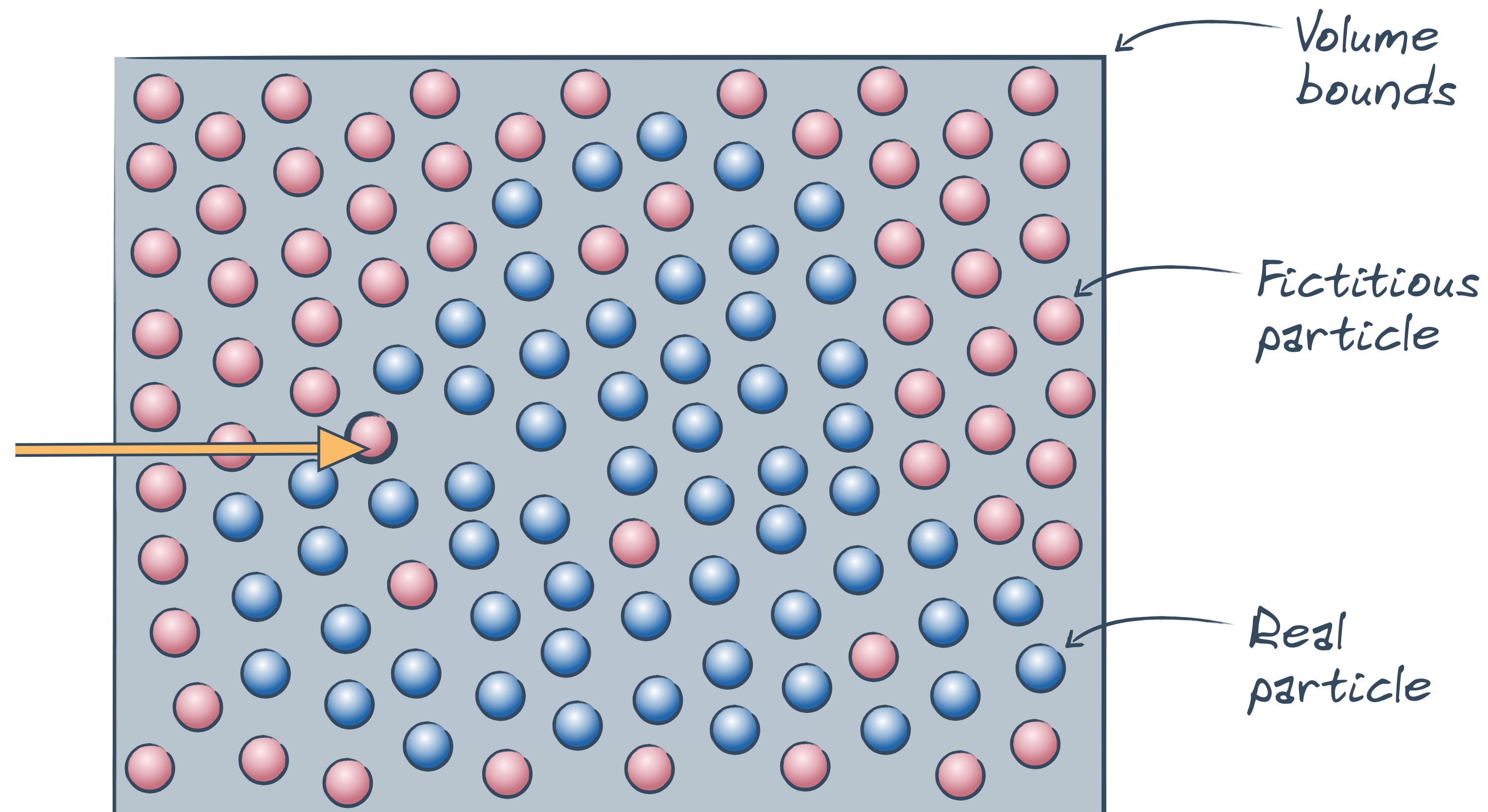
HOMOGENIZATION



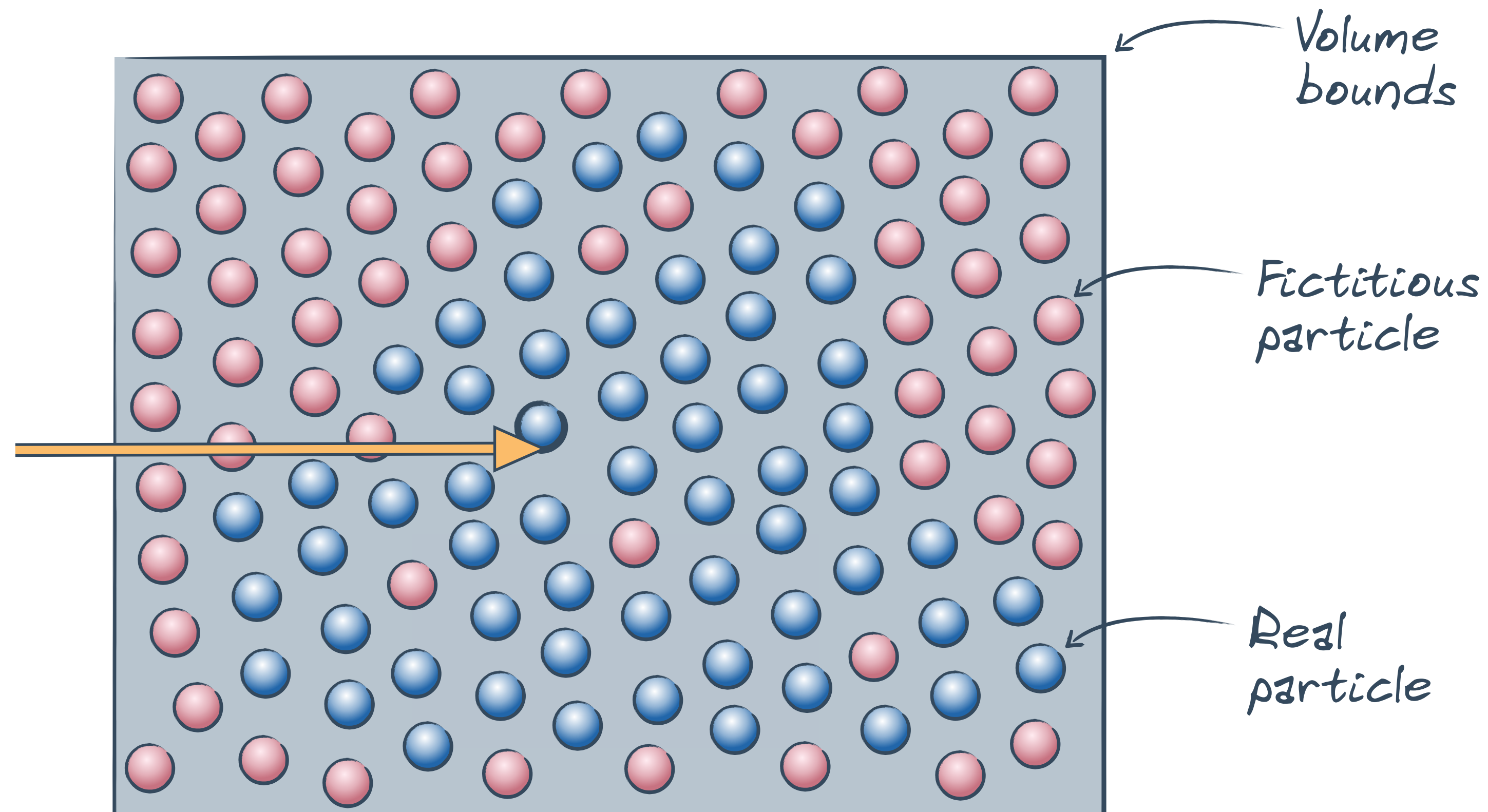
HOMOGENIZATION



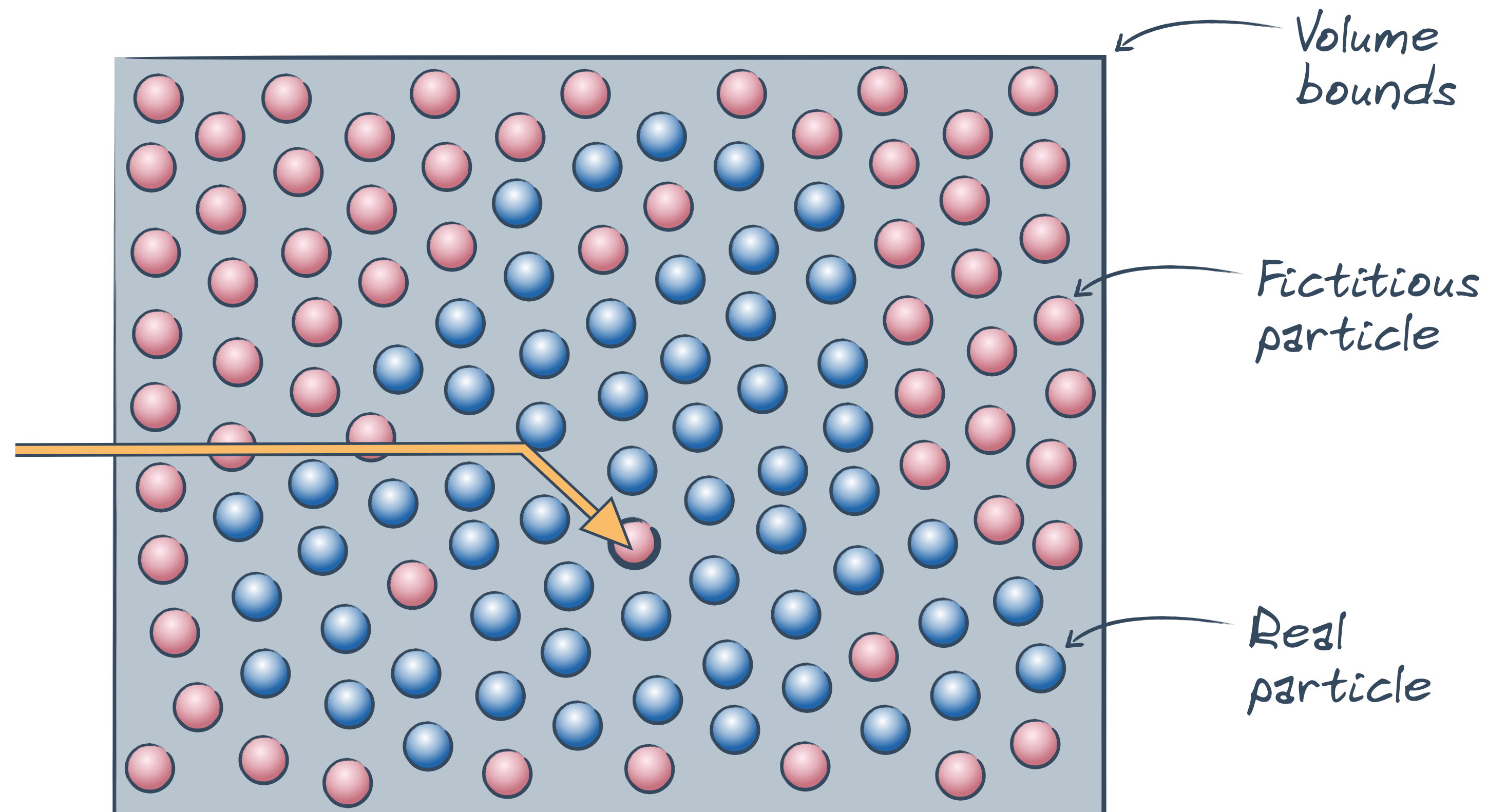
HOMOGENIZATION



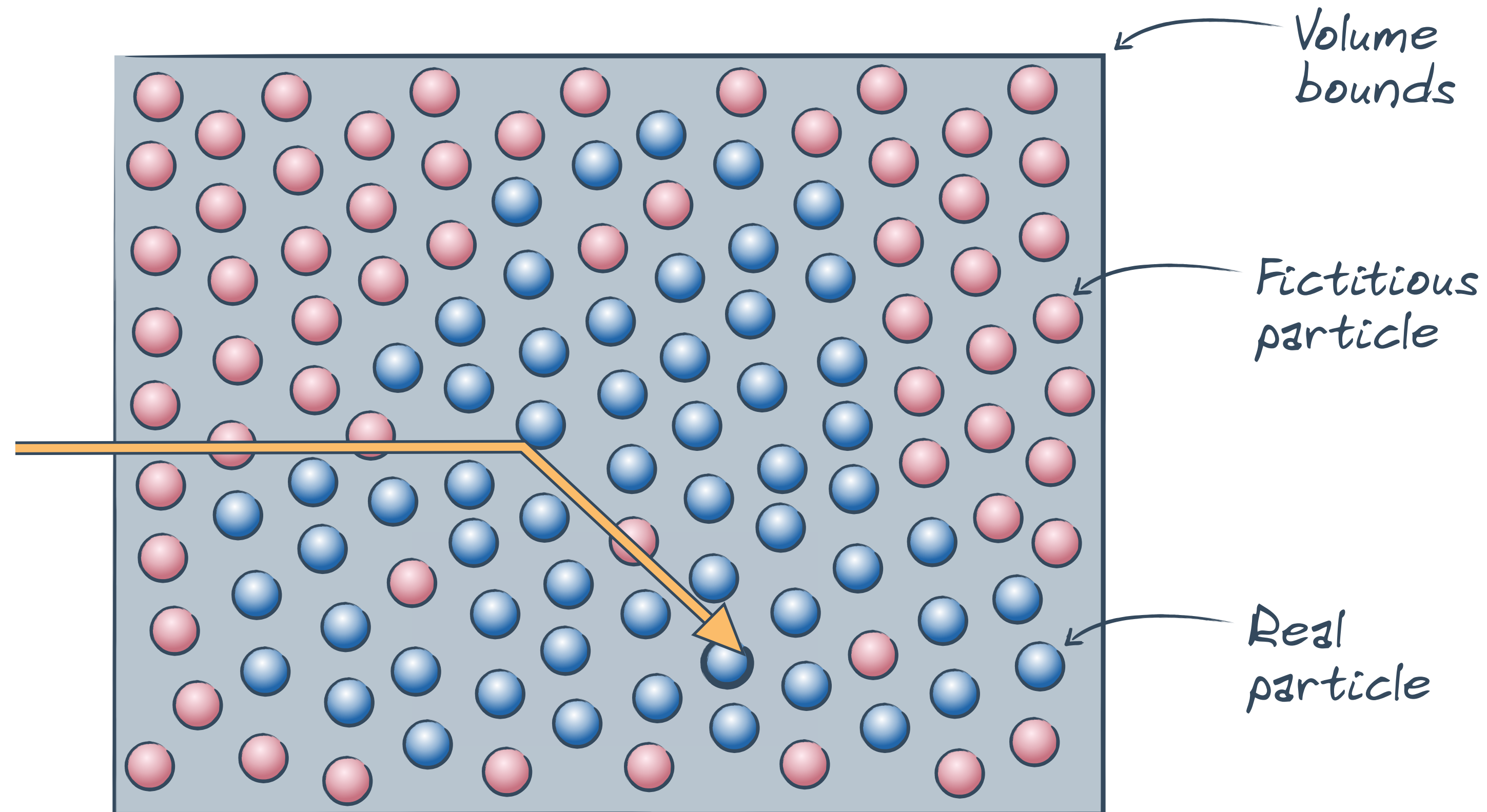
HOMOGENIZATION



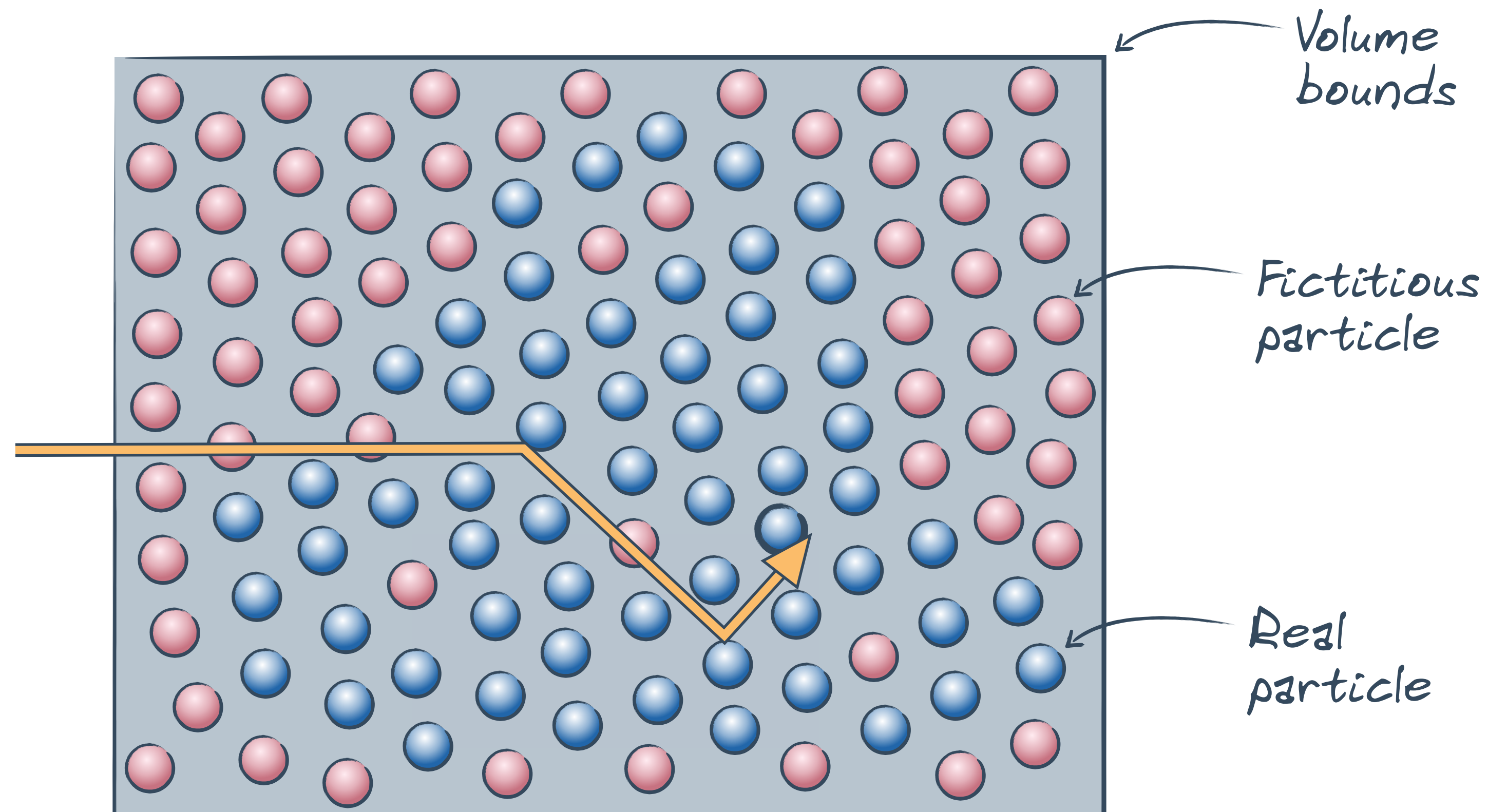
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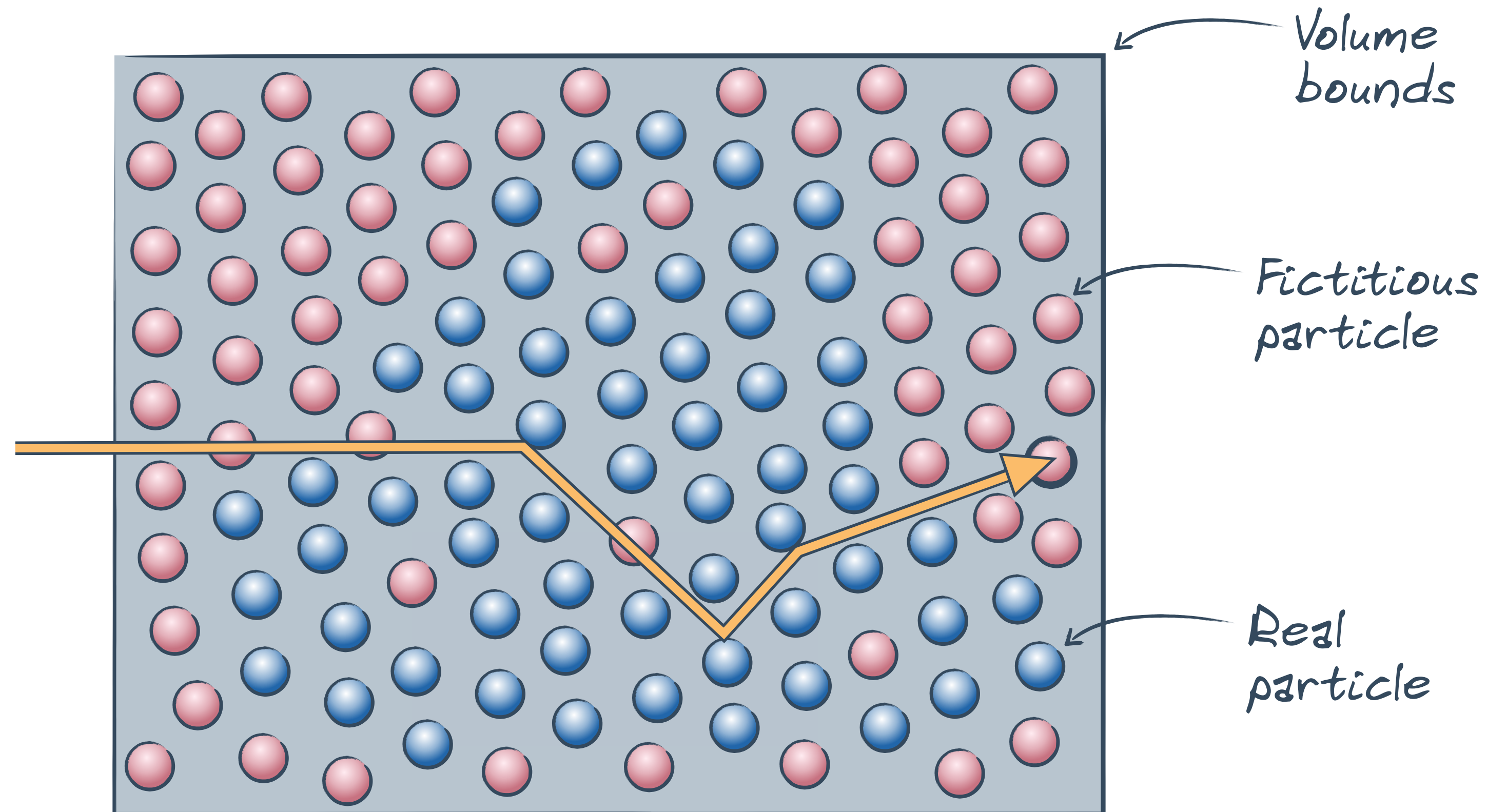
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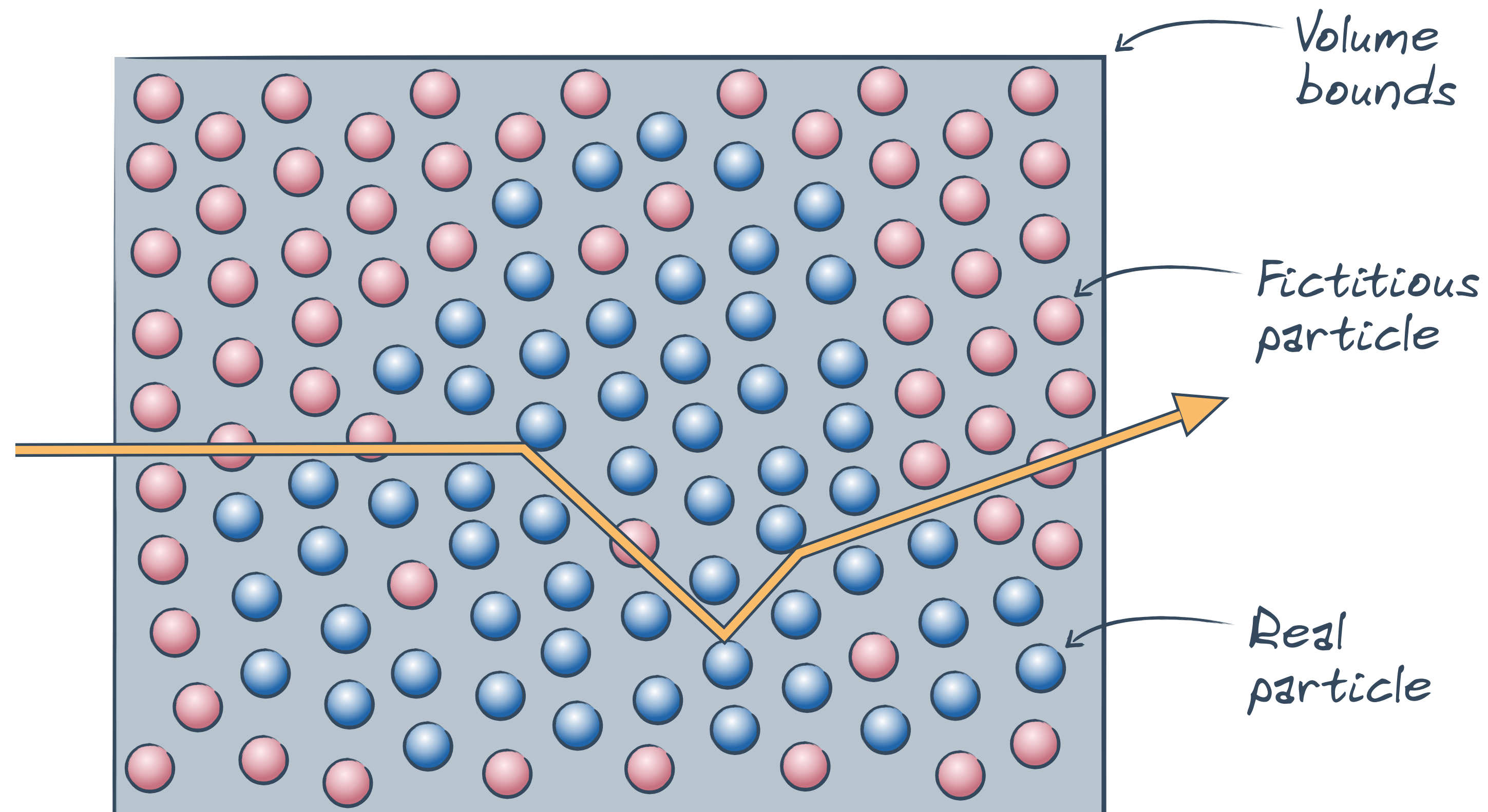
HOMOGENIZATION



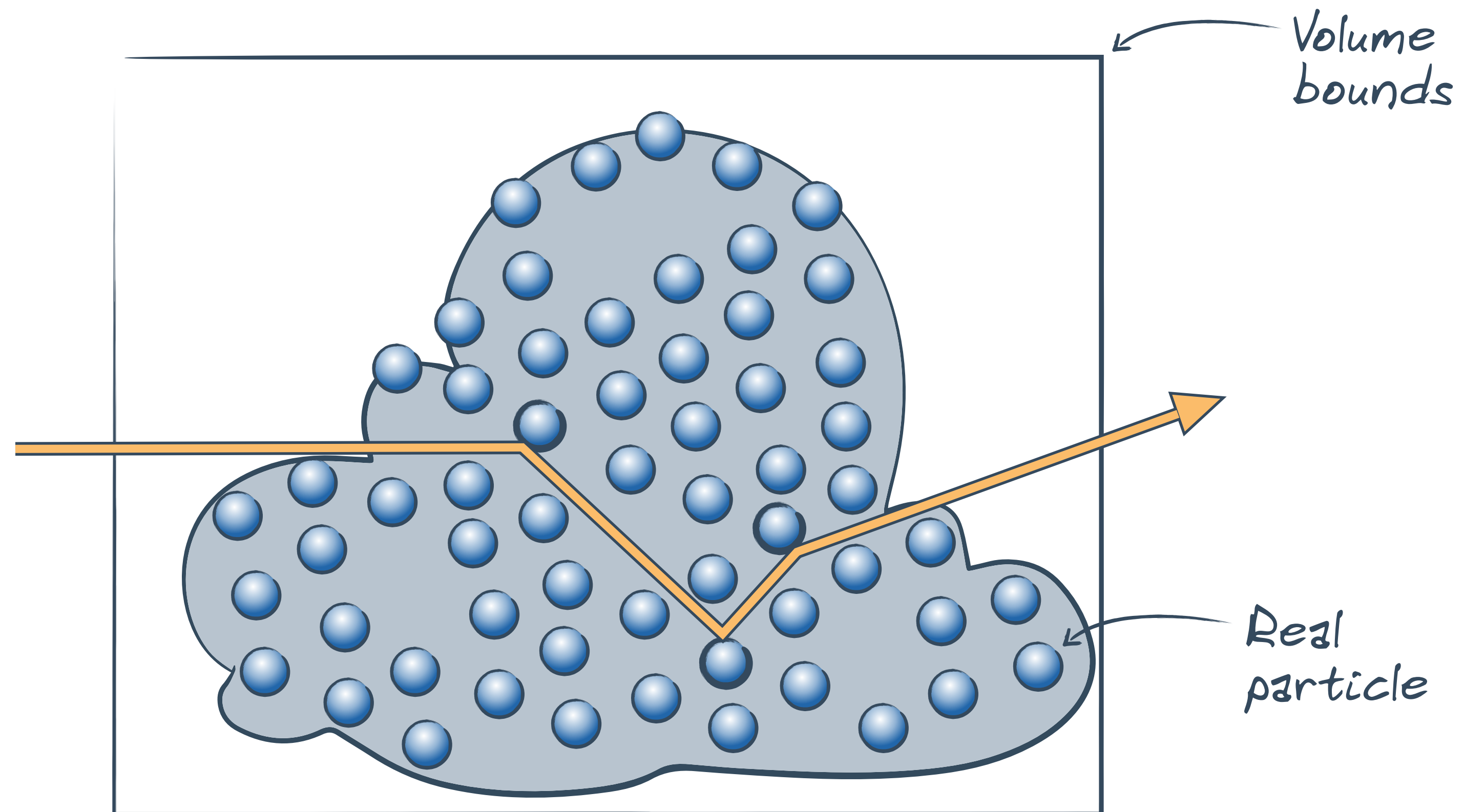
HOMOGENIZATION



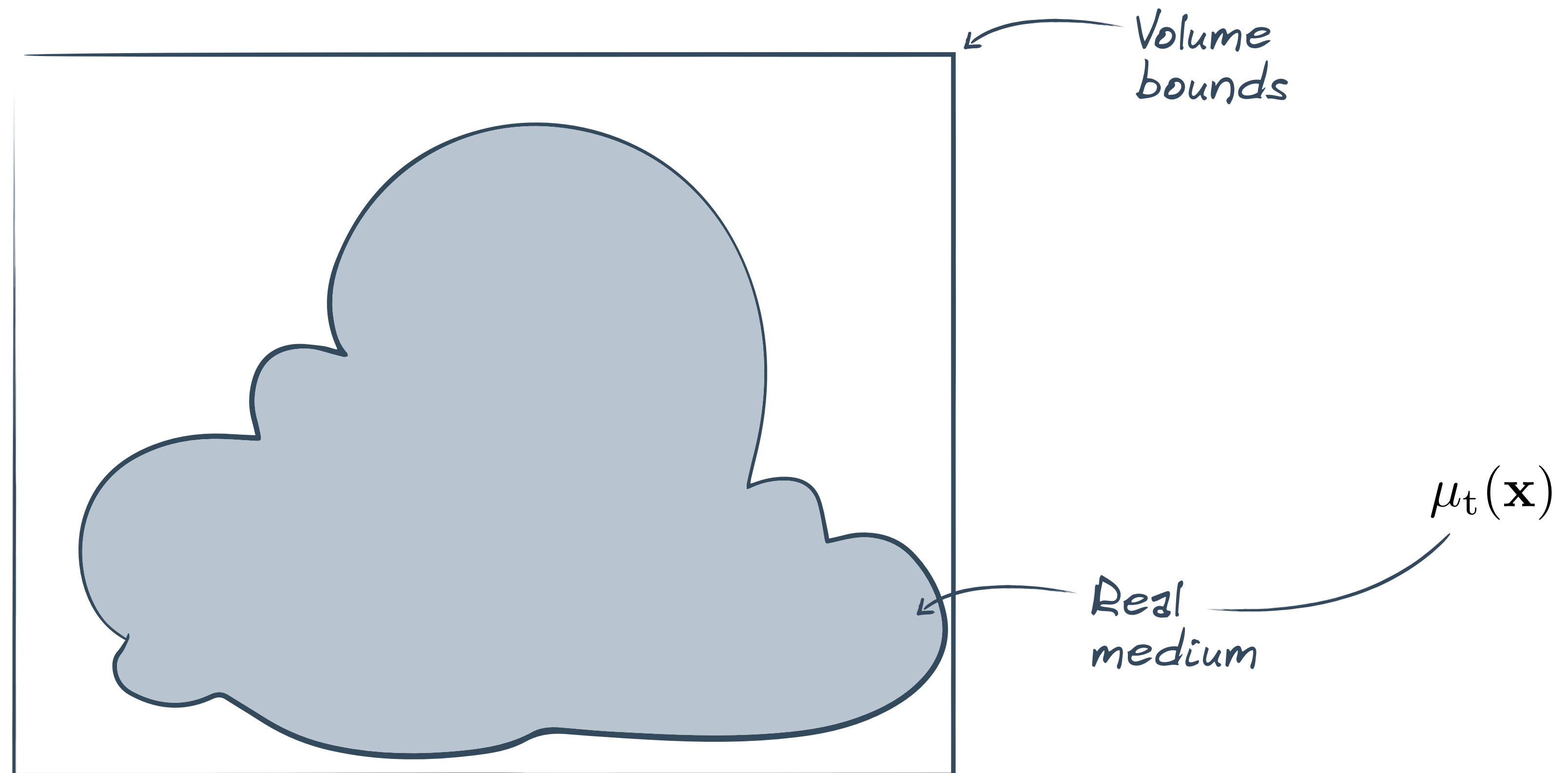
HOMOGENIZATION



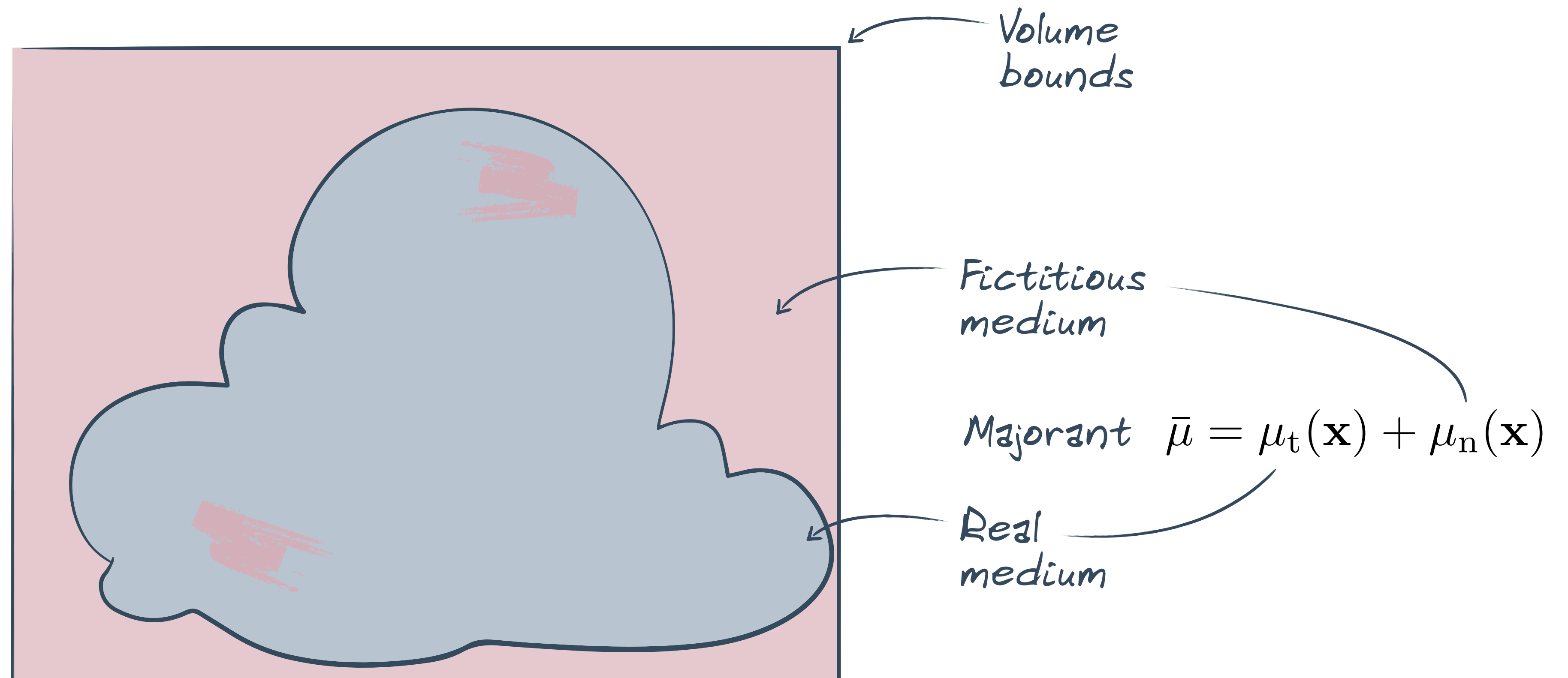
HOMOGENIZATION



STOCHASTIC SAMPLING

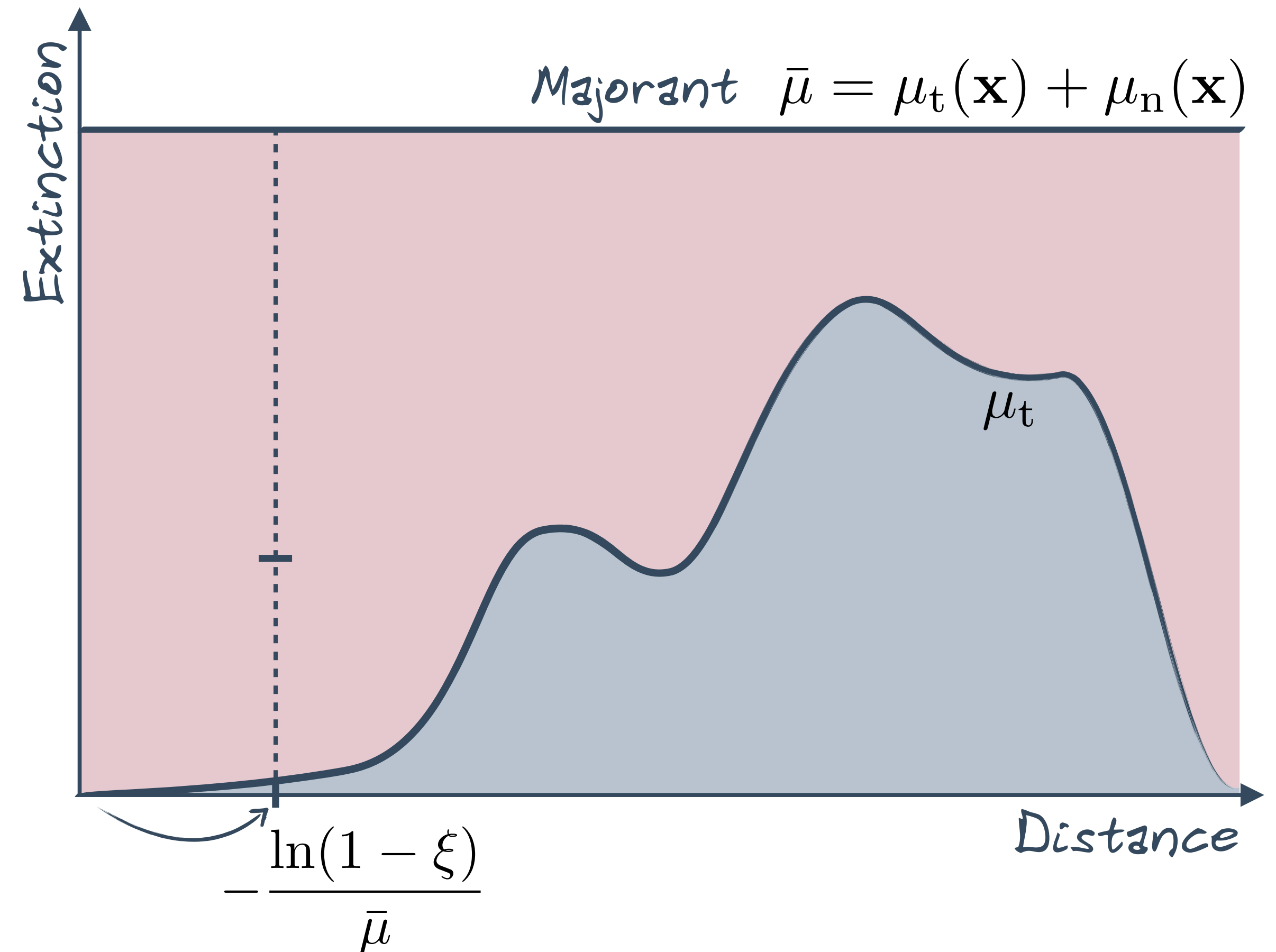
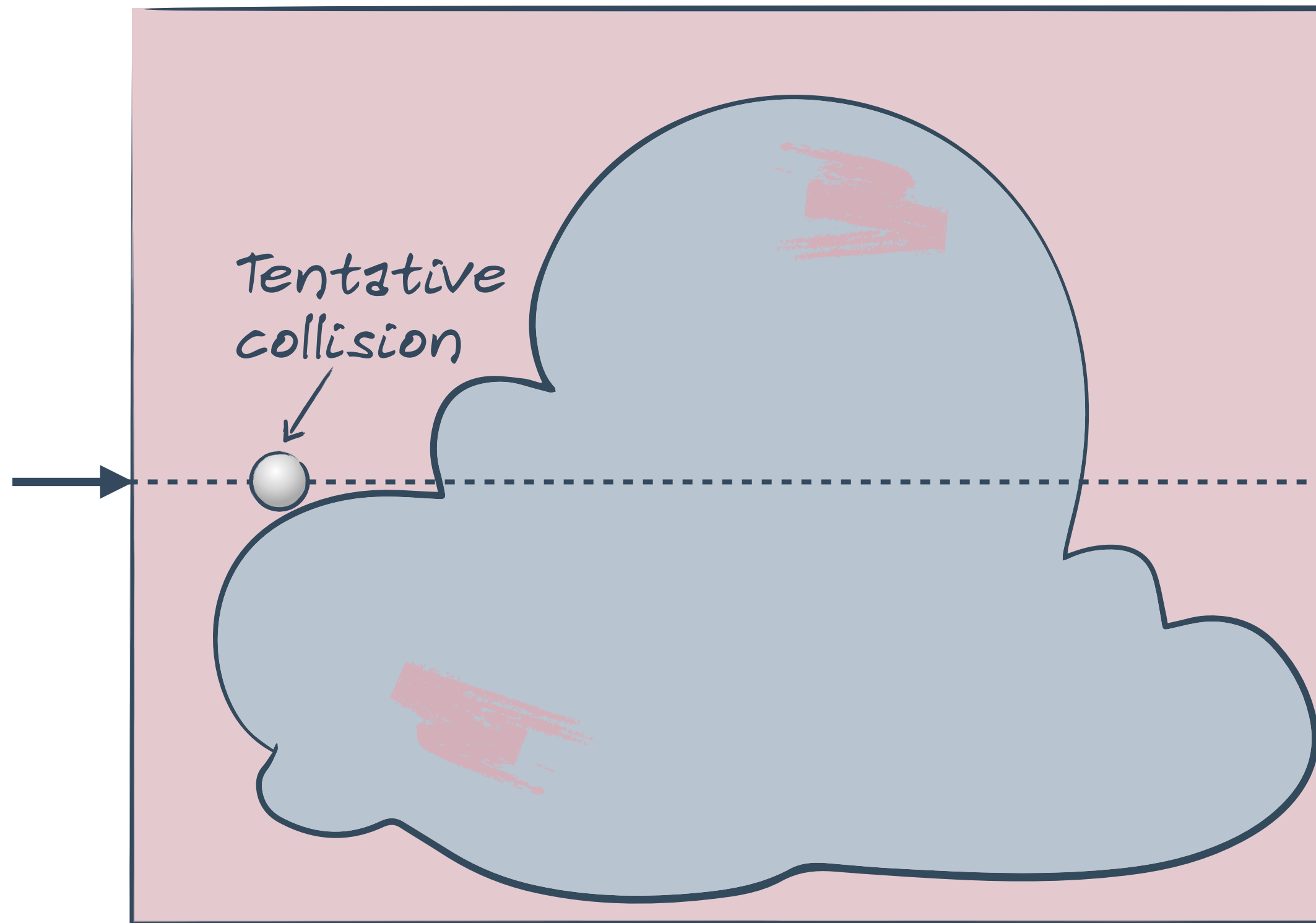


STOCHASTIC SAMPLING



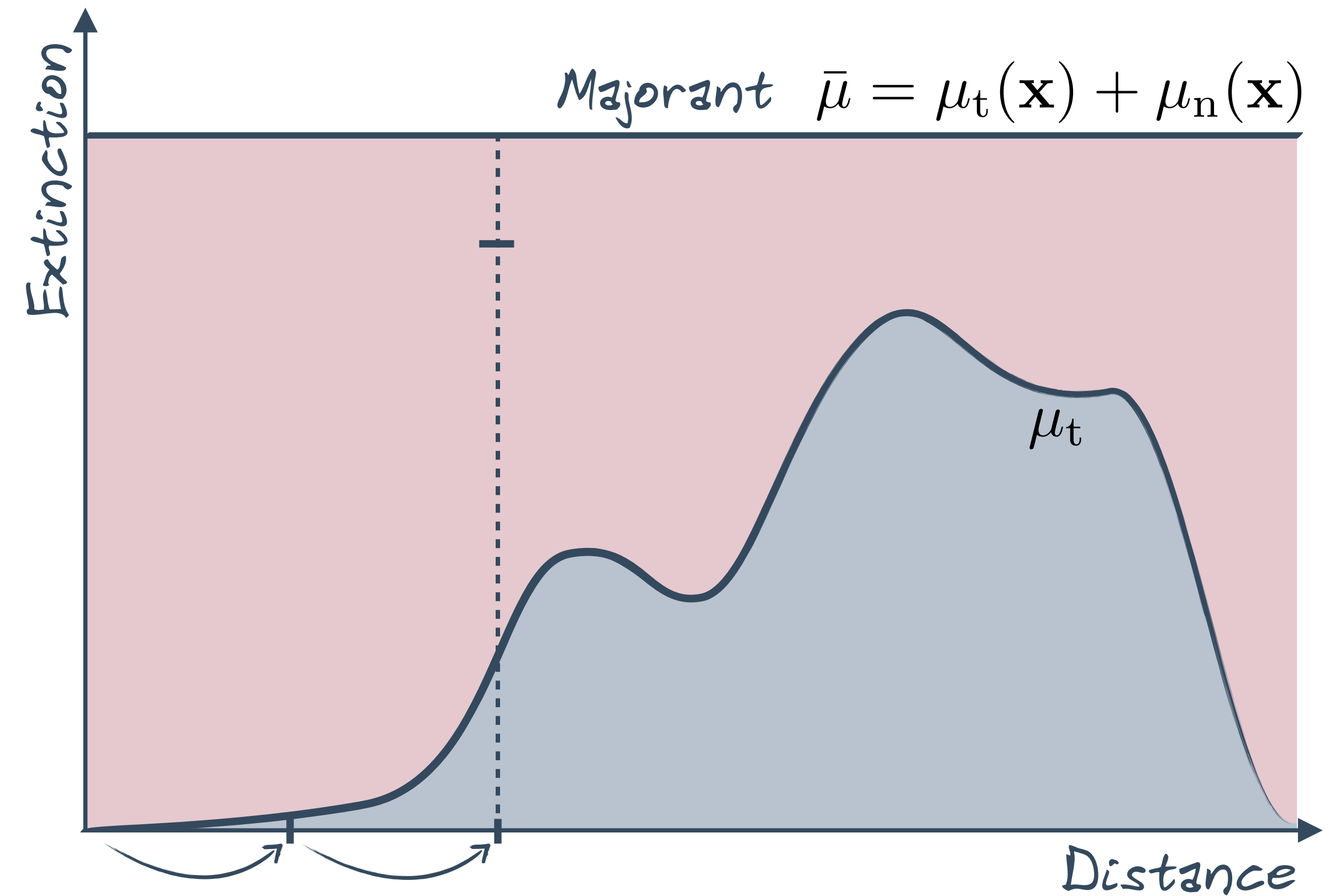
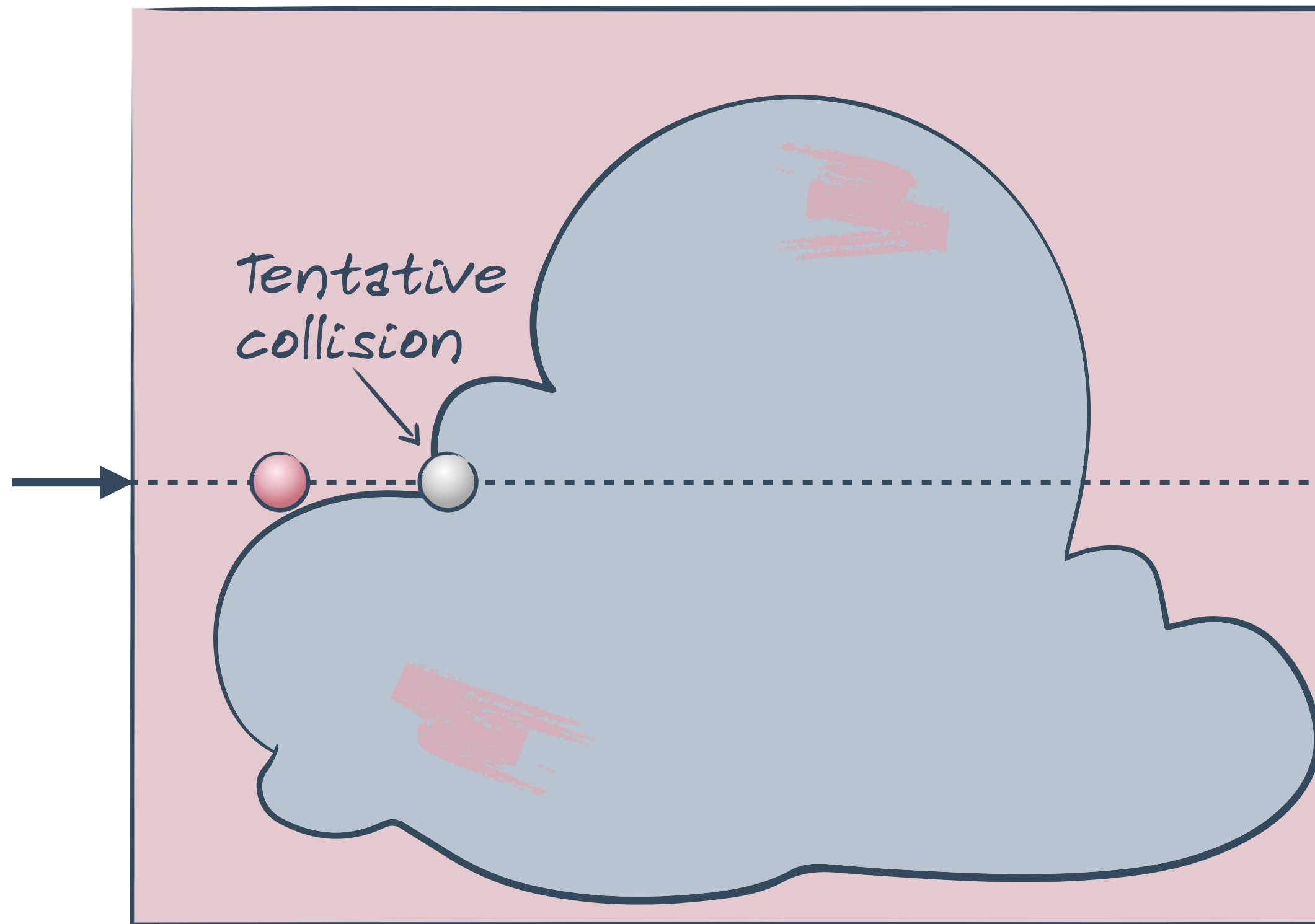
STOCHASTIC SAMPLING

$$P_r(\mathbf{x}) = \frac{\mu_t(\mathbf{x})}{\bar{\mu}} \quad P_n(\mathbf{x}) = \frac{\mu_n(\mathbf{x})}{\bar{\mu}}$$



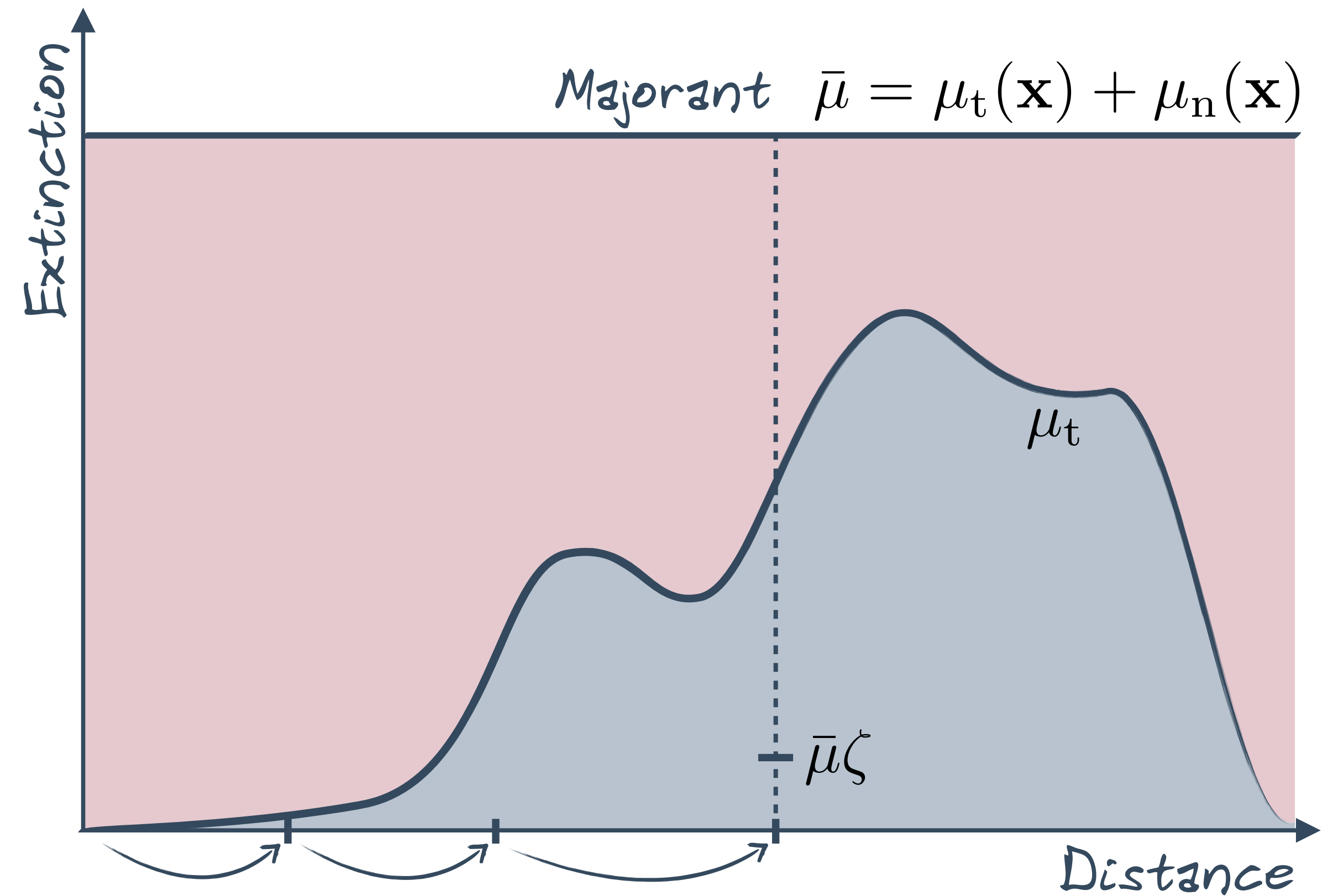
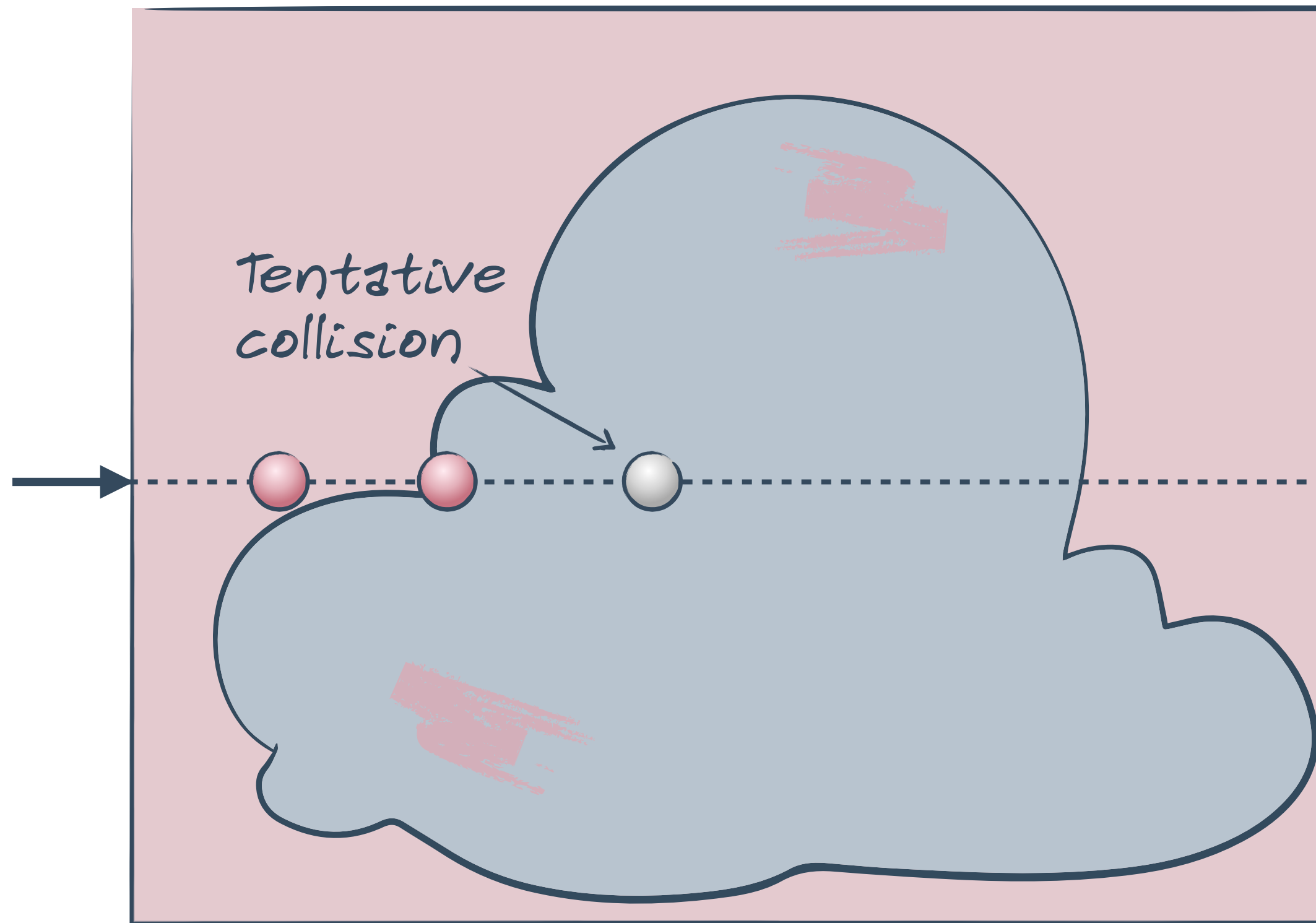
STOCHASTIC SAMPLING

$$P_r(\mathbf{x}) = \frac{\mu_t(\mathbf{x})}{\bar{\mu}} \quad P_n(\mathbf{x}) = \frac{\mu_n(\mathbf{x})}{\bar{\mu}}$$



STOCHASTIC SAMPLING

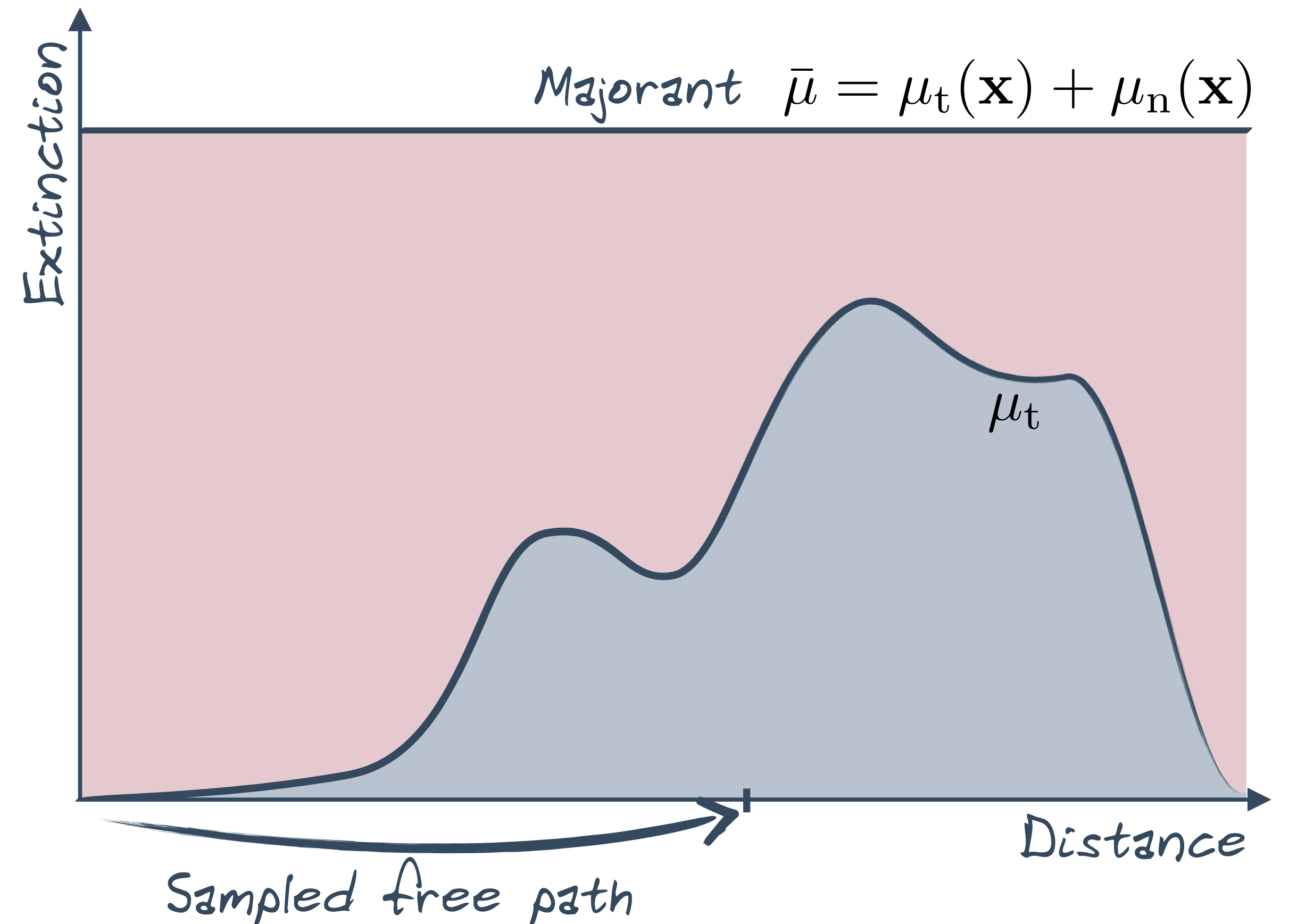
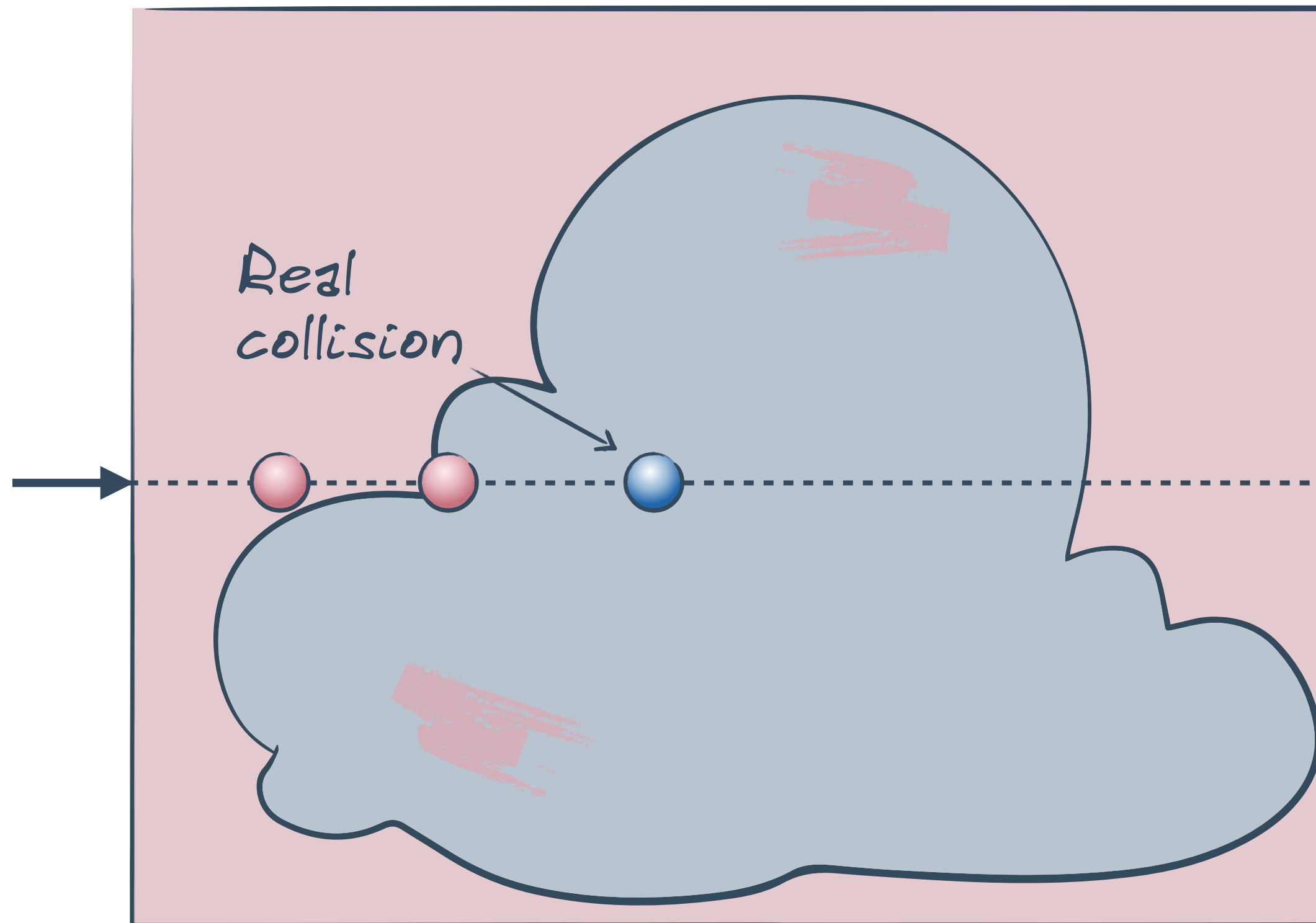
$$P_r(\mathbf{x}) = \frac{\mu_t(\mathbf{x})}{\bar{\mu}} \quad P_n(\mathbf{x}) = \frac{\mu_n(\mathbf{x})}{\bar{\mu}}$$



STOCHASTIC SAMPLING

$$P_r(\mathbf{x}) = \frac{\mu_t(\mathbf{x})}{\bar{\mu}} \quad P_n(\mathbf{x}) = \frac{\mu_n(\mathbf{x})}{\bar{\mu}}$$

Majorant $\bar{\mu} = \mu_t(\mathbf{x}) + \mu_n(\mathbf{x})$



PHYSICALLY-BASED interpretation

- ▶ Correctness motivated by intuitive arguments:
Butcher and Messel [1958, 1960],
Zerby et al. [1961], Bertini [1963],
Woodcock et al. [1965], Skullerud [1968],
...

MATHEMATICAL formalism

- ▶ Integral formulation: Galtier et al. [2013]

DELTA TRACKING

WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

CHANGE OF RADIANCE due to null collisions

$$-\mu_n(\mathbf{x})L(\mathbf{x}, \omega) + \mu_n(\mathbf{x}) \int_{S^2} \delta(\omega - \bar{\omega})L(\mathbf{x}, \bar{\omega}) d\bar{\omega} = 0$$

Losses \leftarrow \rightarrow Gains ("in-scattering")

Cancel each other

CHANGE OF RADIANCE due to null collisions

$$-\mu_n(\mathbf{x})L(\mathbf{x}, \omega) + \mu_n(\mathbf{x}) \int_{S^2} \delta(\omega - \bar{\omega})L(\mathbf{x}, \bar{\omega}) d\bar{\omega} = 0$$

INTEGRAL RTE with null collisions

$$L(\mathbf{x}, \omega) = \int_0^\infty T_{\bar{\mu}}(y) \left[\mu_a(\mathbf{y})L_e(\mathbf{y}, \omega) + \mu_s(\mathbf{y})L_s(\mathbf{y}, \omega) + \underbrace{\mu_n(\mathbf{y})L(\mathbf{y}, \omega)}_{\text{Null-collided radiance}} \right] dy$$

Transmittance through the combined (real+fictitious) medium

INTEGRAL RTE with null collisions

$$L(\mathbf{x}, \omega) = \int_0^\infty T_{\bar{\mu}}(y) \left[\mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) + \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) + \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \right] dy$$

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) + \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) + \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \right]$$

Probabilistic evaluation



RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) + \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) + \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \right]$$

*Probabilistic evaluation
using Russian roulette*

$$\langle f(x) \rangle_P = \begin{cases} \frac{f(x)}{P(x)} & \text{if accepted} \\ 0 & \text{otherwise} \end{cases}$$

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

*Probabilistic evaluation
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RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Represents an entire family of
(weighted) trackers that all solve RTE!

Delta tracking is just one specific instance.

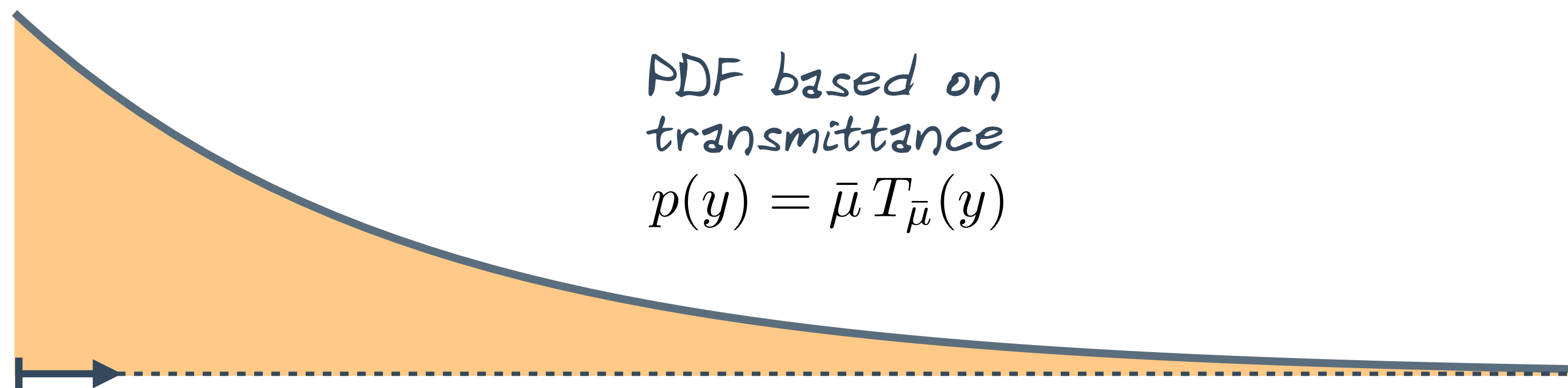
...see the report or
Galtier et al. [2013]
for complete derivation

WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

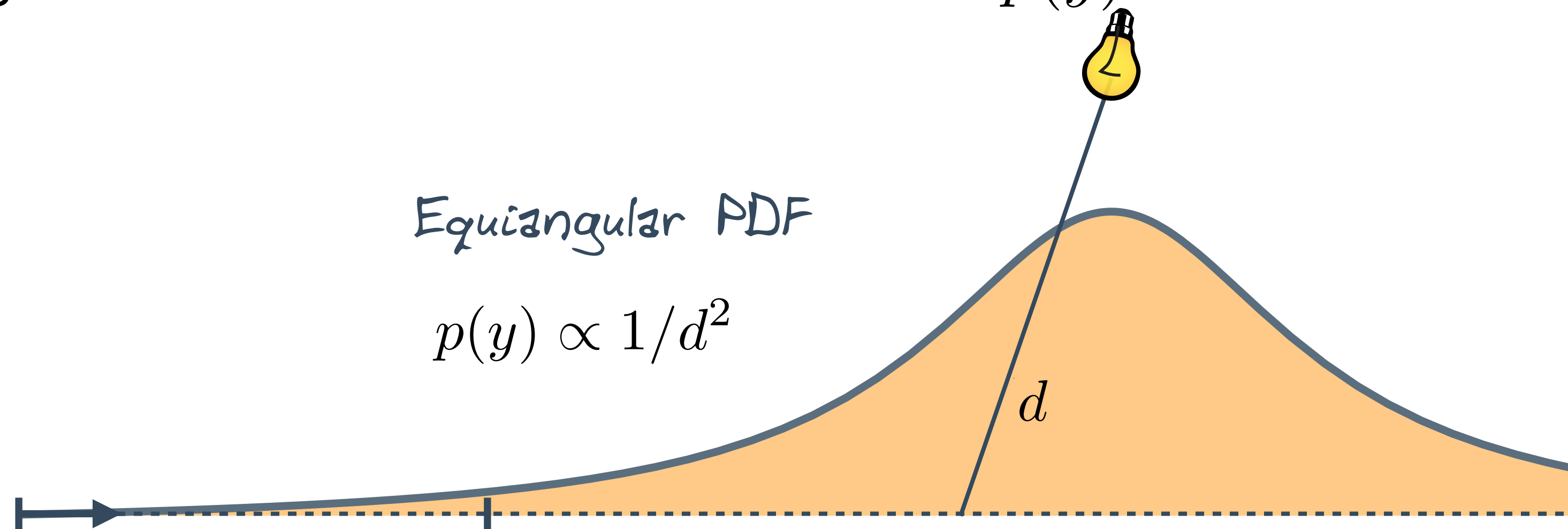


WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

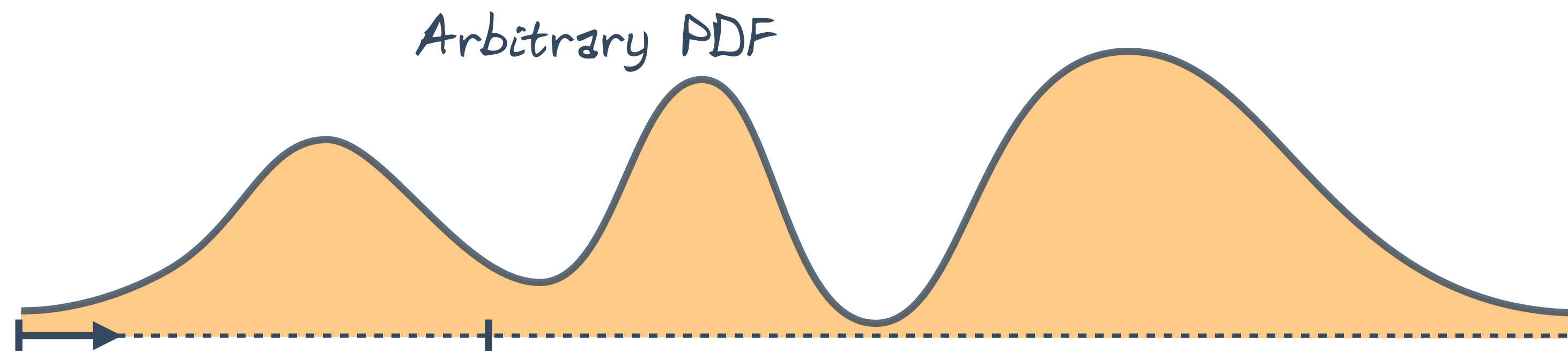


WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$



WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES P_a, P_s, P_n

Delta tracking

$$P_a = \frac{\mu_a}{\bar{\mu}} \quad P_s = \frac{\mu_s}{\bar{\mu}} \quad P_n = \frac{\mu_n}{\bar{\mu}}$$



WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

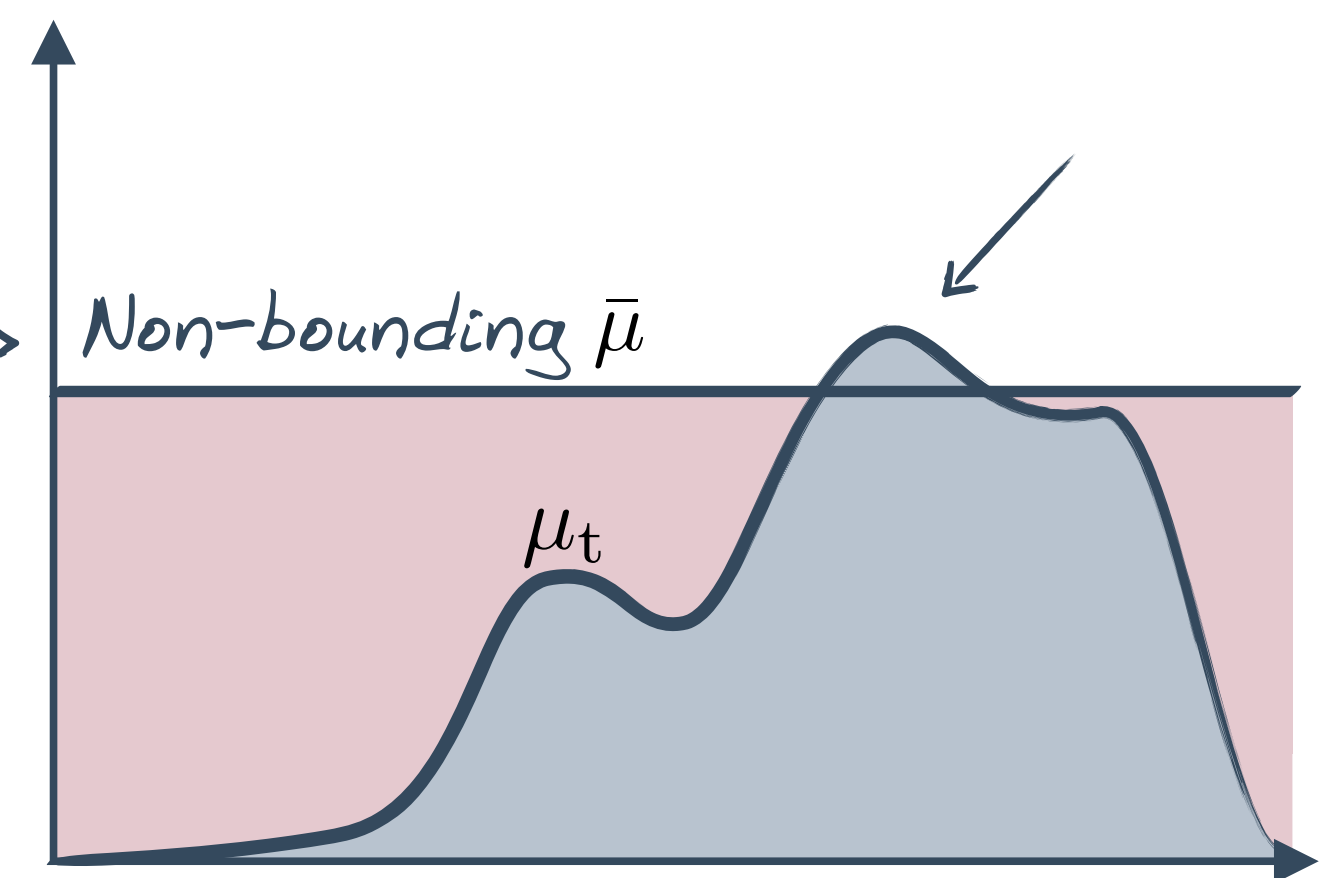
$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES P_a, P_s, P_n

Weighted tracking that handles

$$P_a = \frac{\mu_a}{\mu_t + |\mu_n|} \quad P_s = \frac{\mu_s}{\mu_t + |\mu_n|} \quad P_n = \frac{|\mu_n|}{\mu_t + |\mu_n|}$$



WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

Degrees of freedom: **DISTANCE SAMPLING** $p(y)$

INTERACTION PROBABILITIES P_a, P_s, P_n

Disabled absorption/emission sampling

$$P_a = 0 \quad P_s = \frac{\mu_s}{\mu_s + |\mu_n|} \quad P_n = \frac{|\mu_n|}{\mu_s + |\mu_n|}$$



WEIGHTED (DELTA) TRACKING

RTE ESTIMATOR with null collisions

$$\langle L(\mathbf{x}, \omega) \rangle = \frac{T_{\bar{\mu}}(y)}{p(y)} \left[\langle \mu_a(\mathbf{y}) L_e(\mathbf{y}, \omega) \rangle_{P_a} + \langle \mu_s(\mathbf{y}) L_s(\mathbf{y}, \omega) \rangle_{P_s} + \langle \mu_n(\mathbf{y}) L(\mathbf{y}, \omega) \rangle_{P_n} \right]$$

WEIGHT due to multiple null collisions:

$$\prod_{i=1}^{k-1} \frac{T_{\bar{\mu}}(y_i)}{p(y_i)} \frac{\mu_n(y_i)}{P_n(y_i)}$$



WEIGHTED (DELTA) TRACKING

- ▶ Integral framework for null-collision algorithms
[Galtier et al. 2013]
- ▶ Handling of non-bounding “majorants”
[Cramer 1978, Galtier et al. 2013, Eymet et al. 2013, Novák et al. 2014, Szirmay-Kalos et al. 2017, Kutz et al. 2017, Szirmay-Kalos et al. 2018]
- ▶ Improved transmittance estimation
[Cramer 1978, Novák et al. 2014—Ratio tracking]
- ▶ Sample splitting
[Eymet et al. 2013], [Szirmay-Kalos et al. 2017—Single vs. Double particle model]
- ▶ Spectral tracking
[Kutz et al. 2017]

WEIGHTED (DELTA) TRACKING

SUMMARY

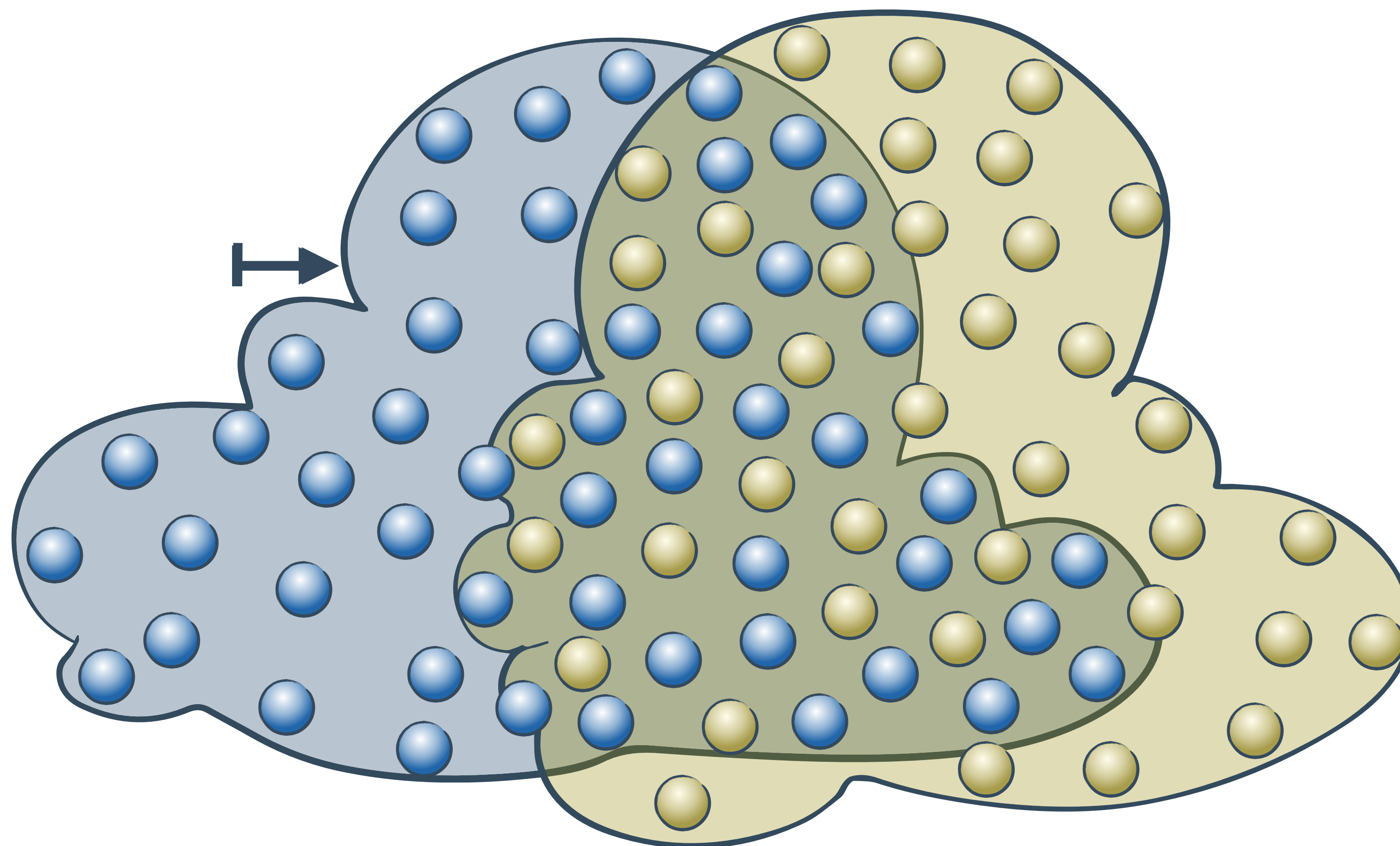
- ▶ Non-analog tracker
- ▶ Distance distribution differs from free-path distribution, but...
distribution of **WEIGHTED** distance samples is **IDENTICAL** to free-path distribution
- ▶ Allows handling non-bounding “majorants”
- ▶ Enables reducing variance by adjusting:
 - ▶ distance sampling of tentative collisions
 - ▶ collision probabilities

DELTA TRACKING

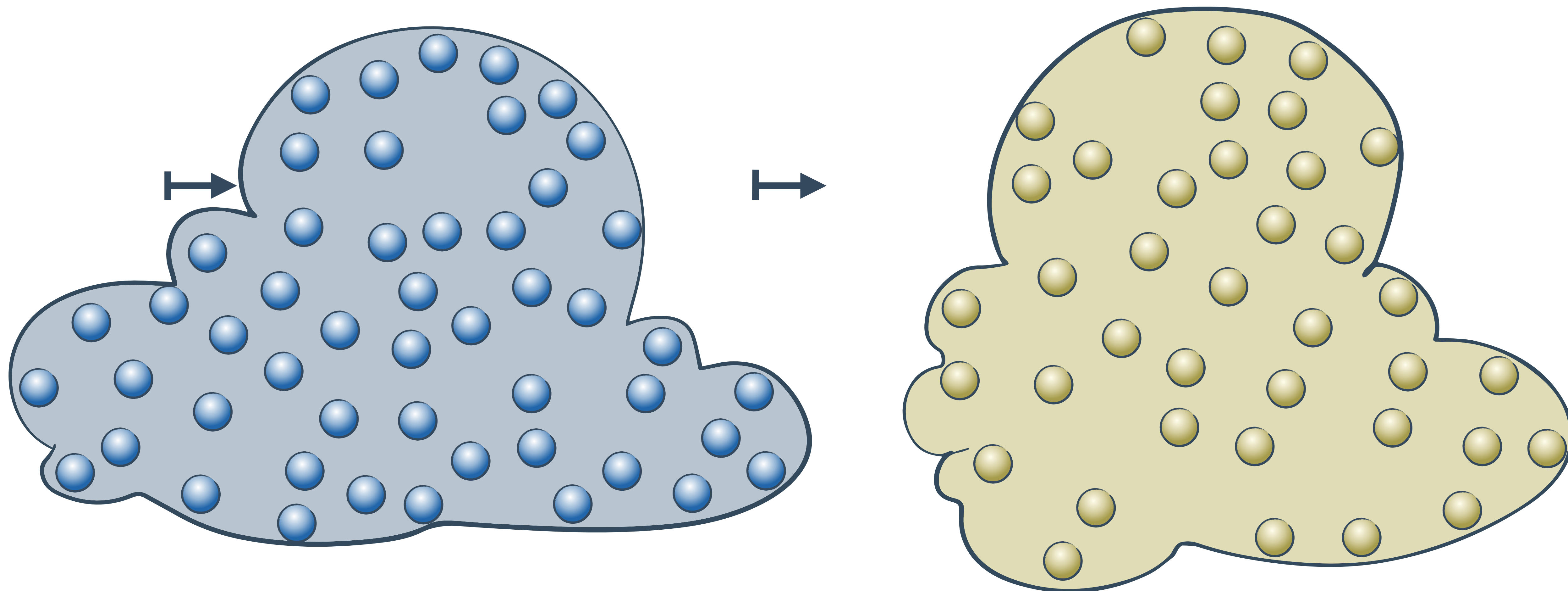
WEIGHTED (DELTA) TRACKING

DECOMPOSITION TRACKING

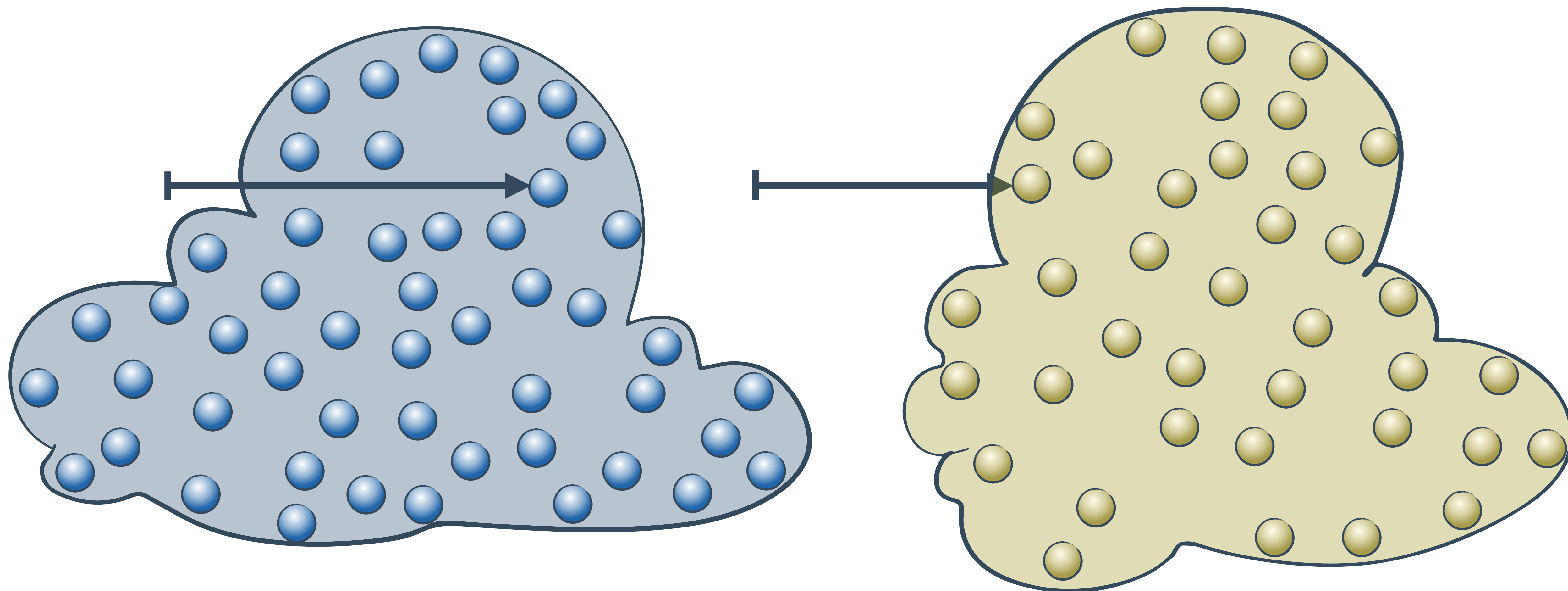
OVERLAPPING VOLUMES



OVERLAPPING VOLUMES



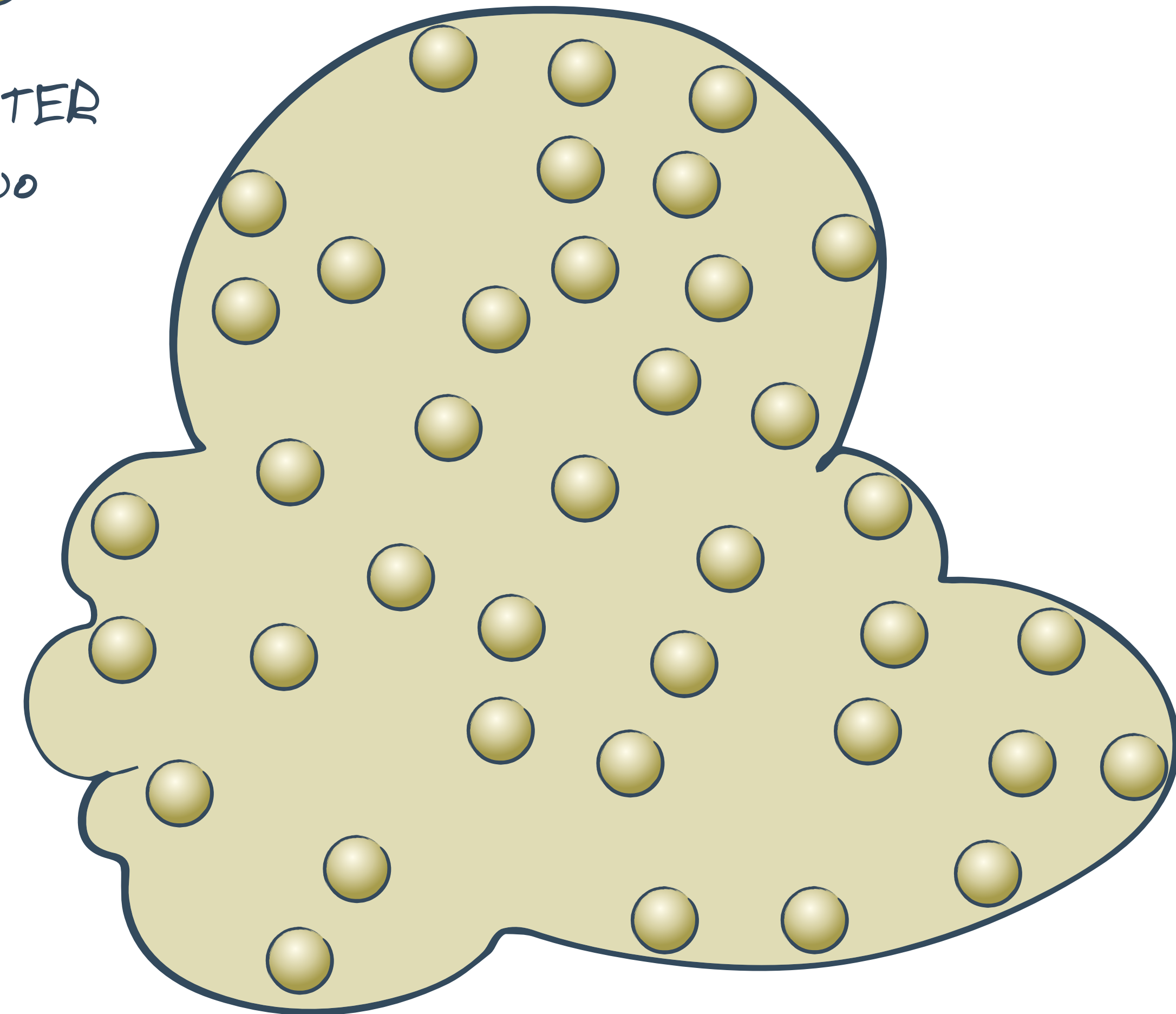
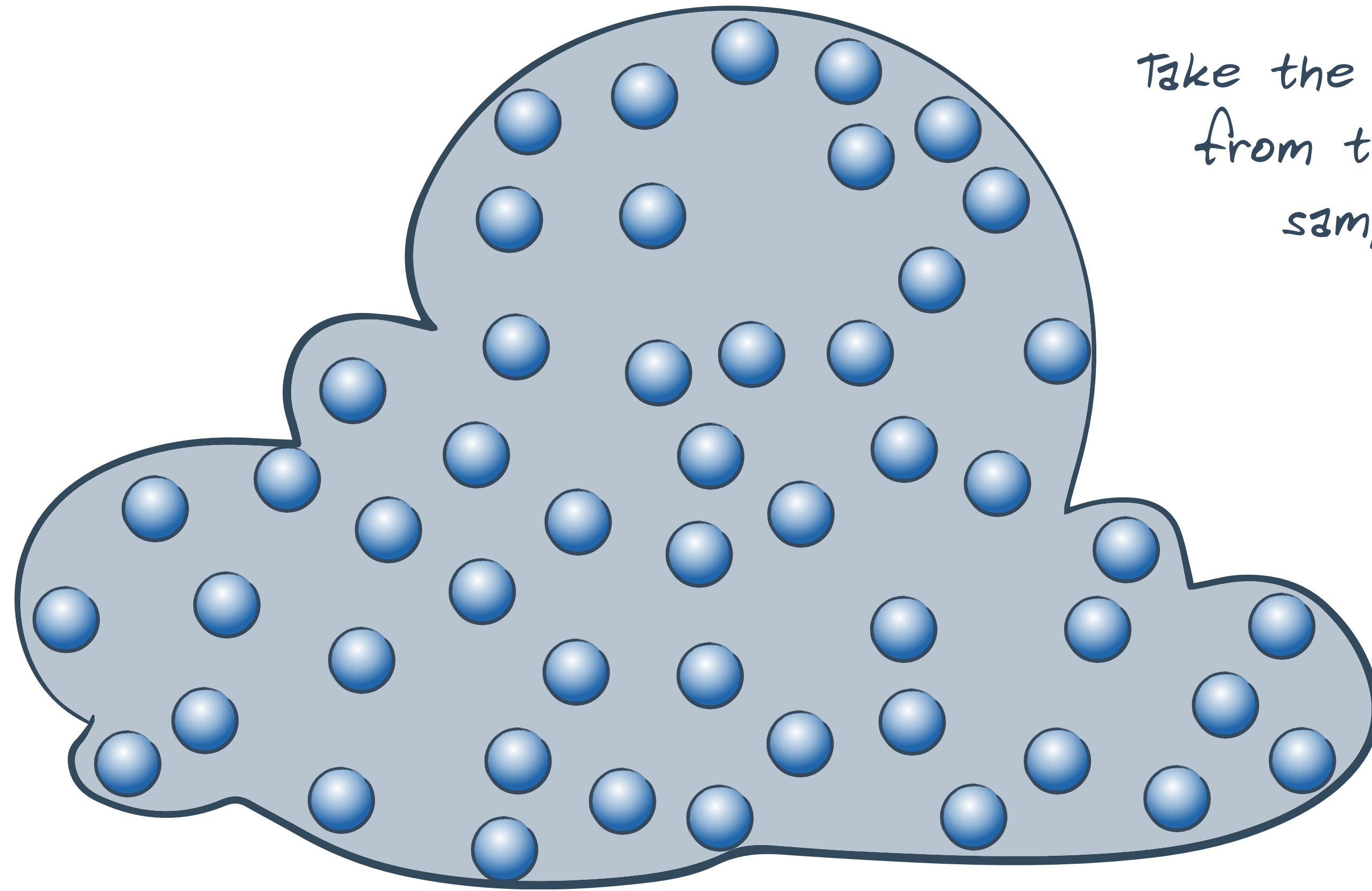
OVERLAPPING VOLUMES



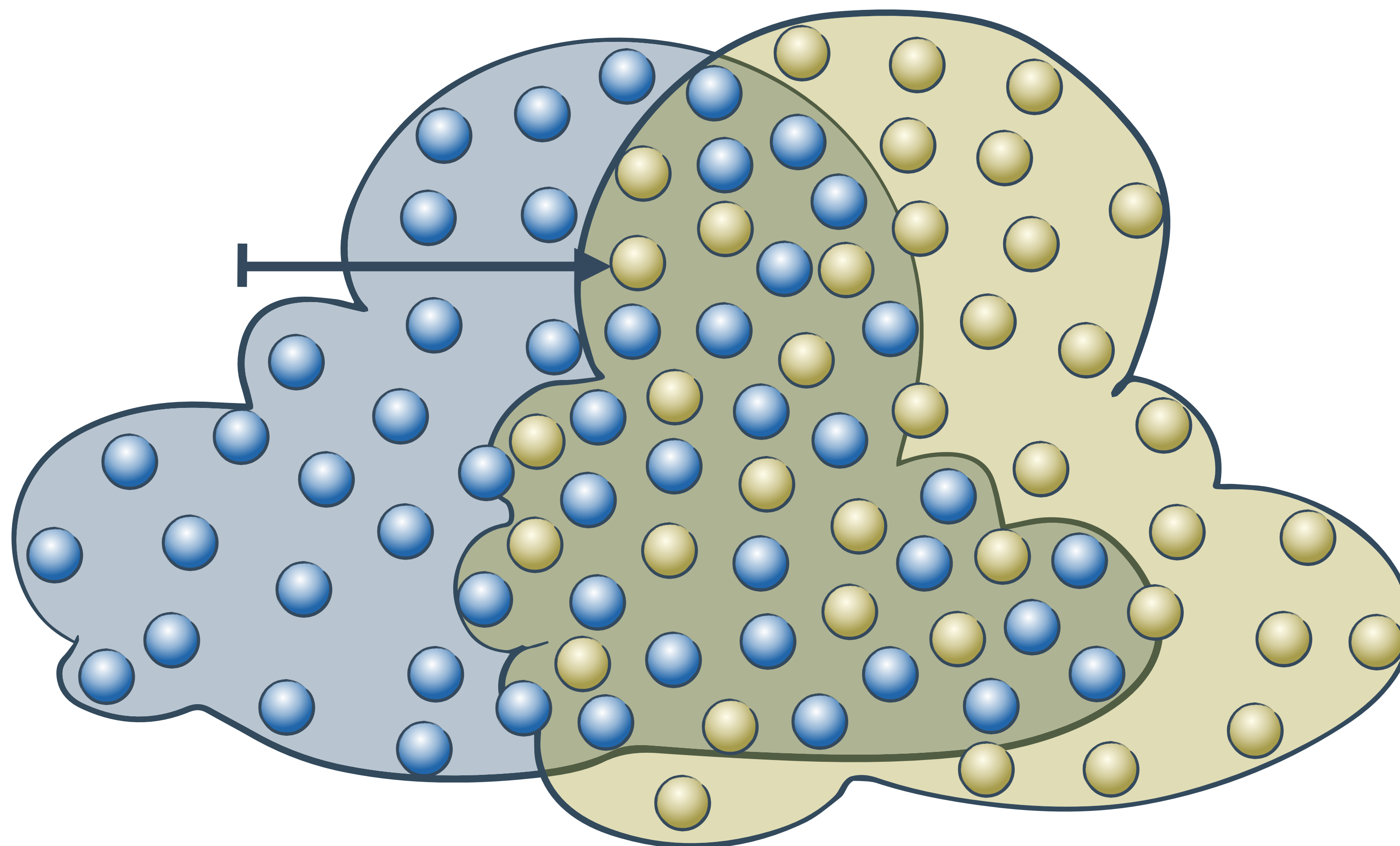
OVERLAPPING VOLUMES



Take the **SHORTER**
from the two
samples



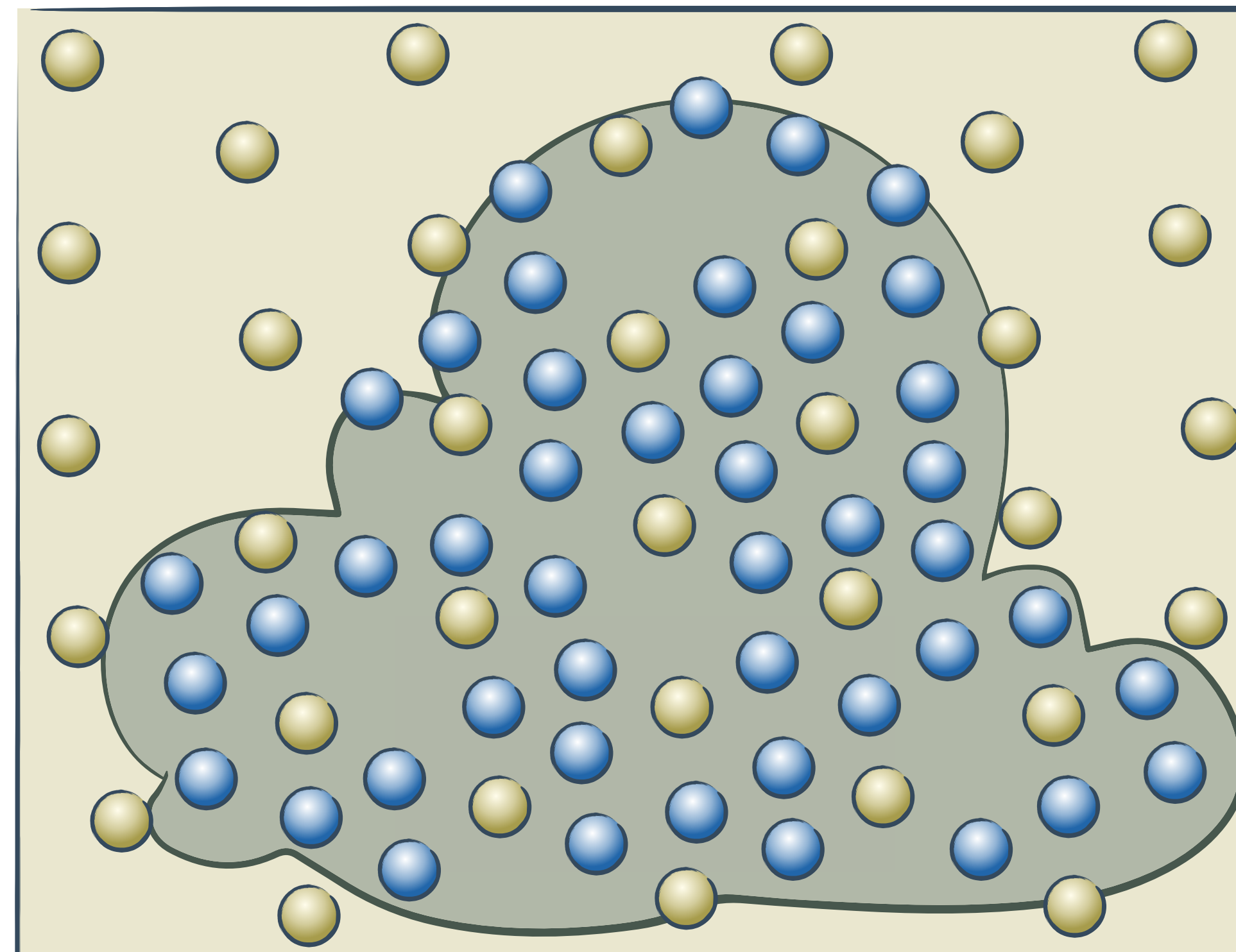
OVERLAPPING VOLUMES



DECOMPOSITION TRACKING

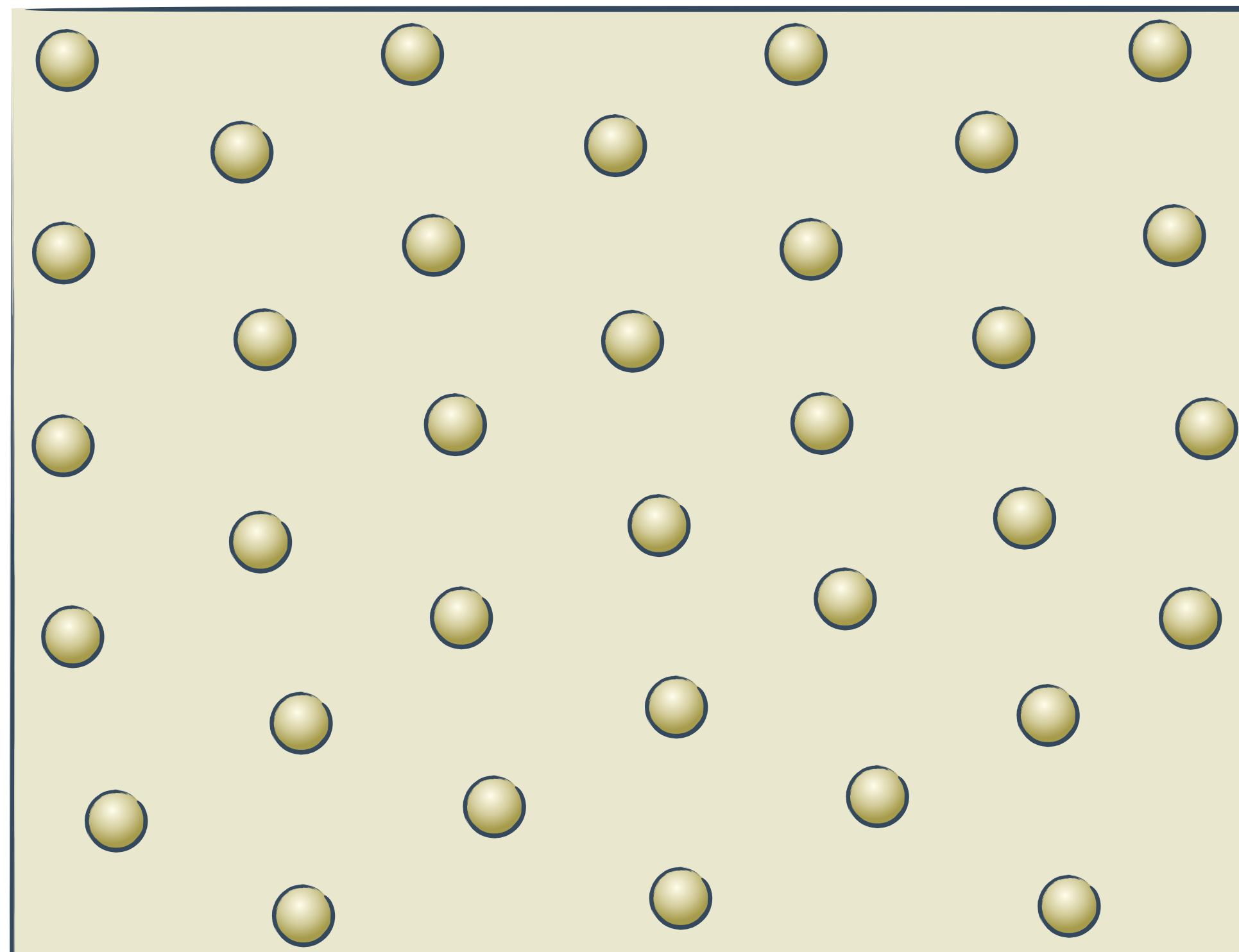
Accelerate free-path sampling by reducing expensive extinction evaluations

- ▶ [Kutz et al. 2017]

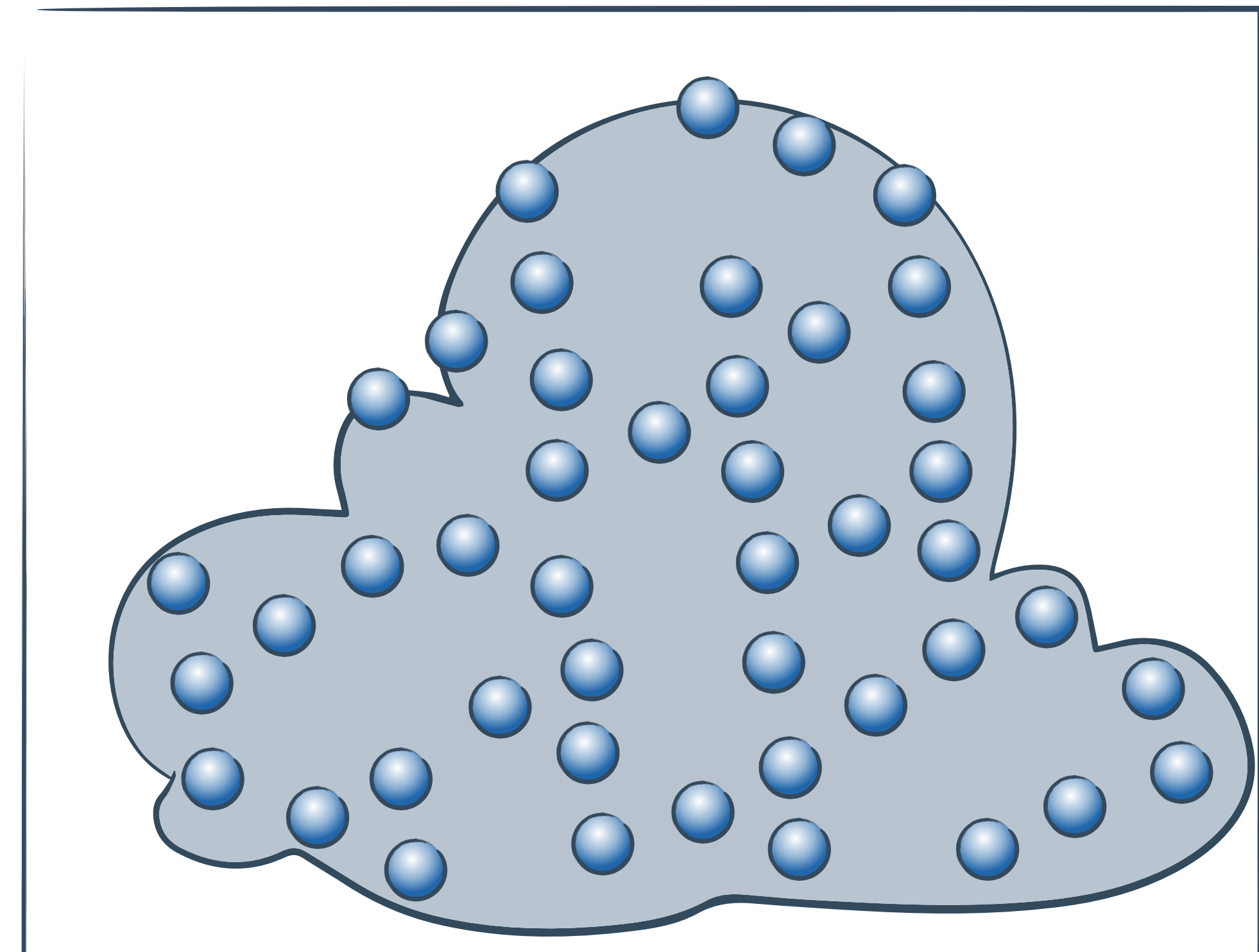


DECOMPOSITION TRACKING

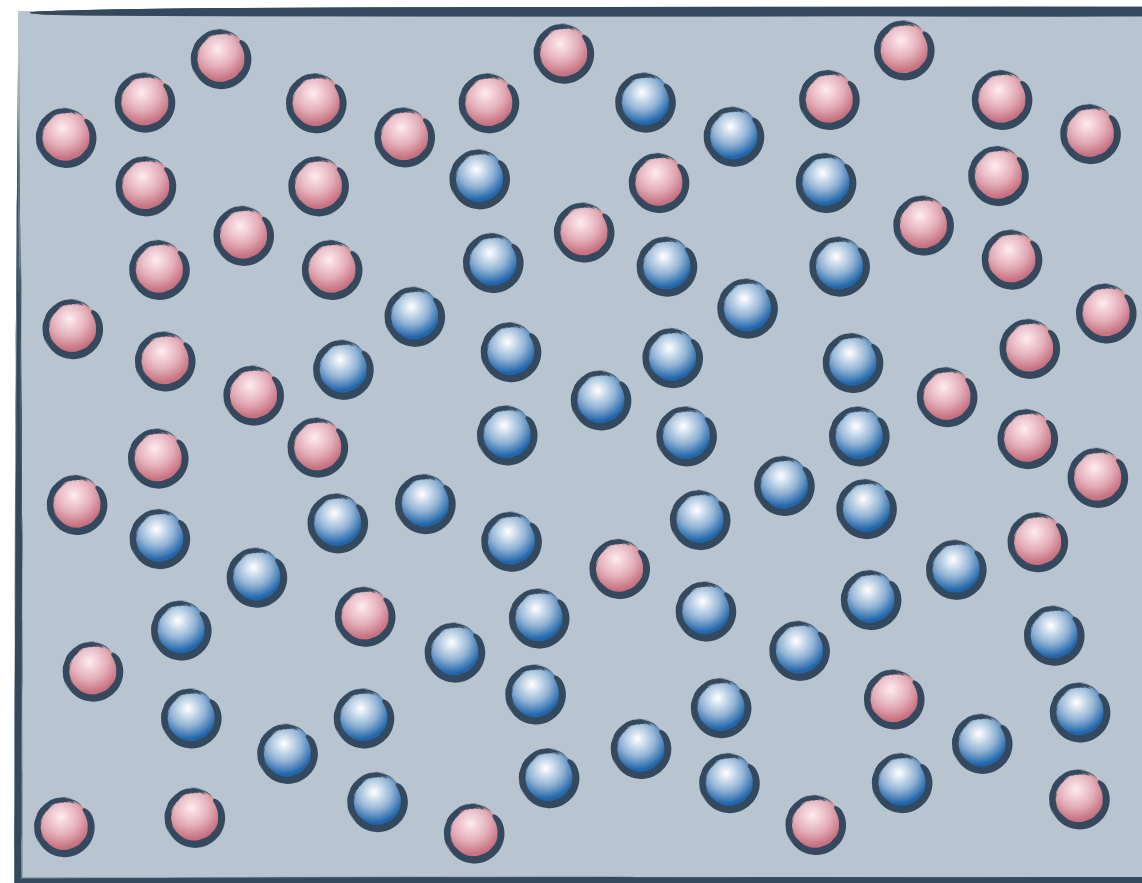
*(Piecewise-) HOMOGENEOUS
component*



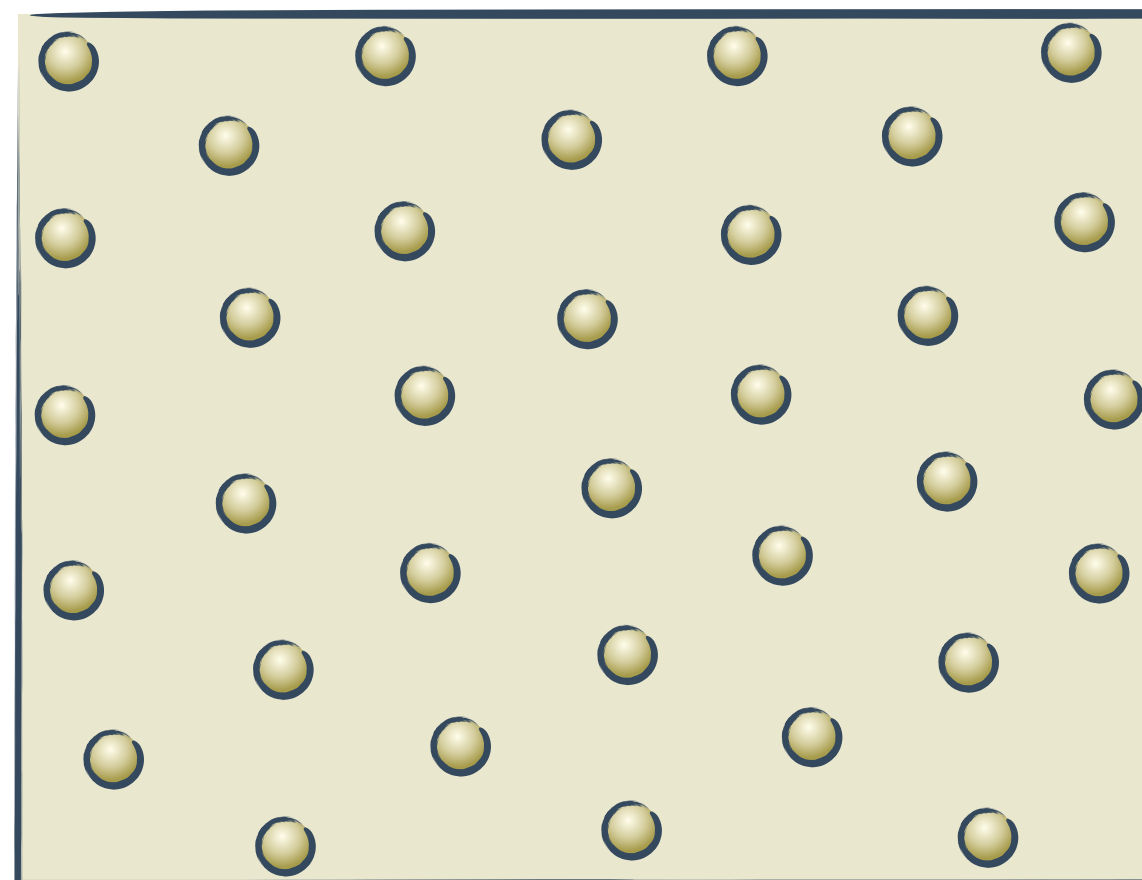
*HETEROGENEOUS
component*



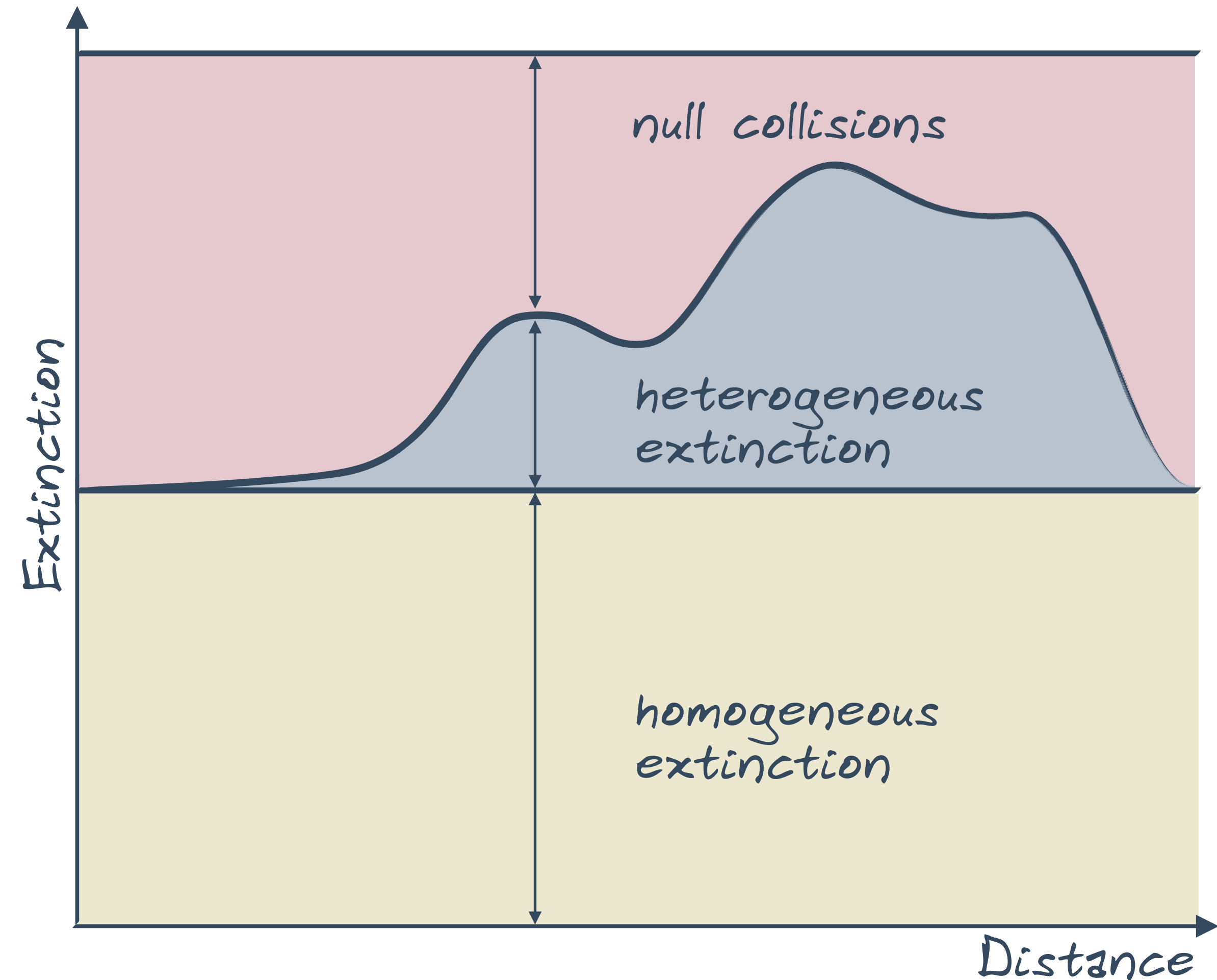
DECOMPOSITION TRACKING



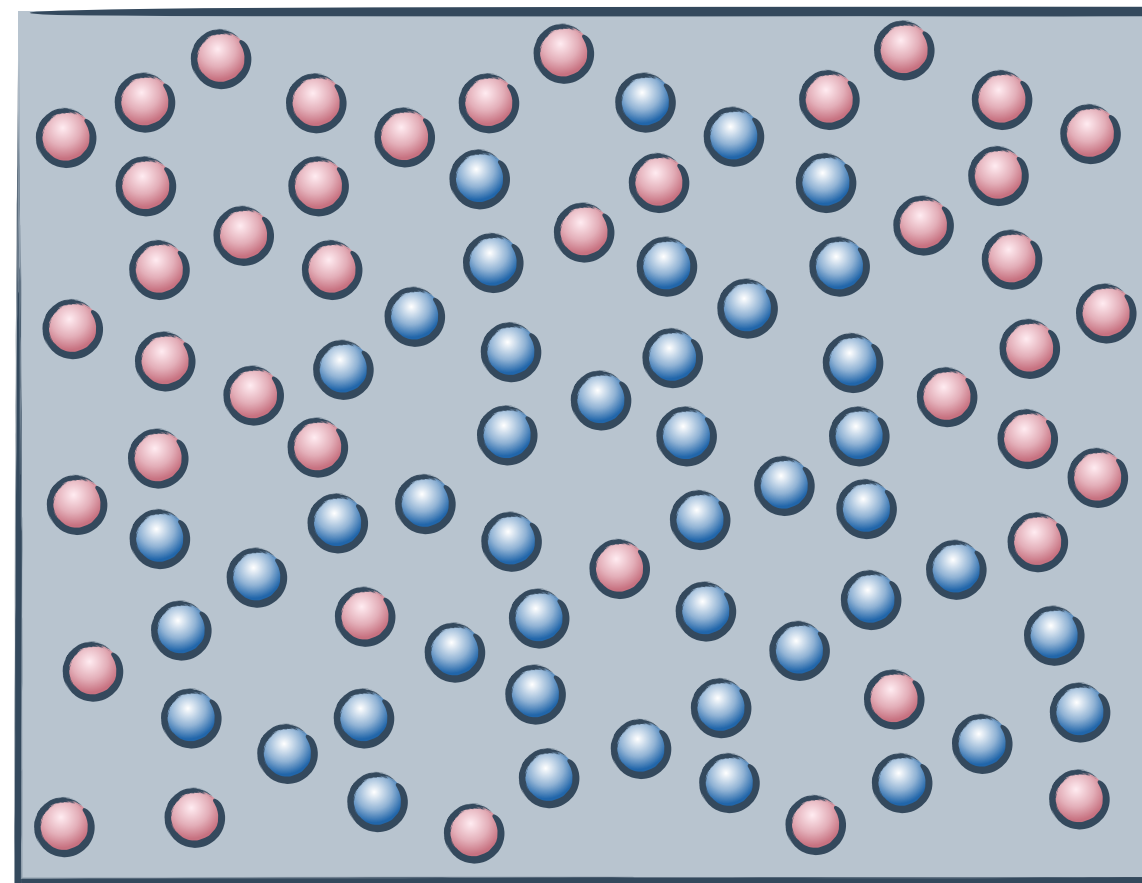
HETEROGENEOUS
component



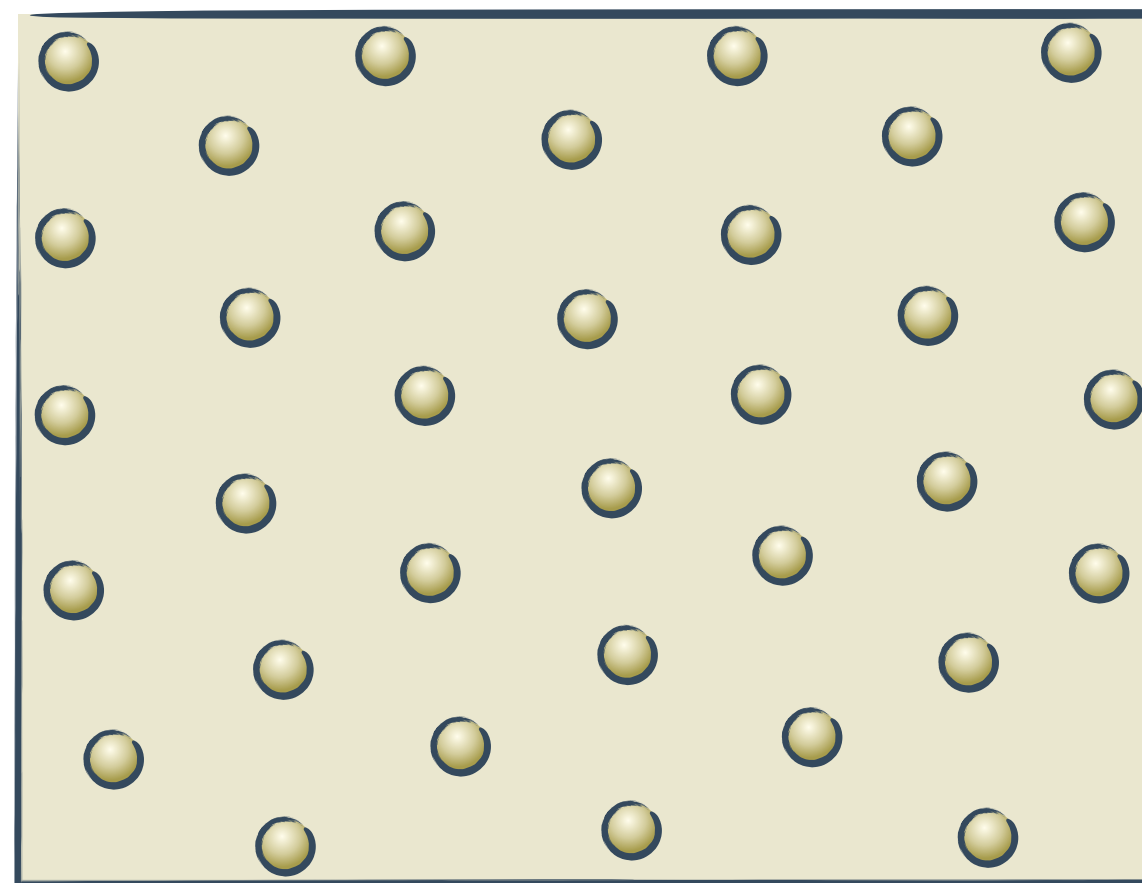
HOMOGENEOUS
component



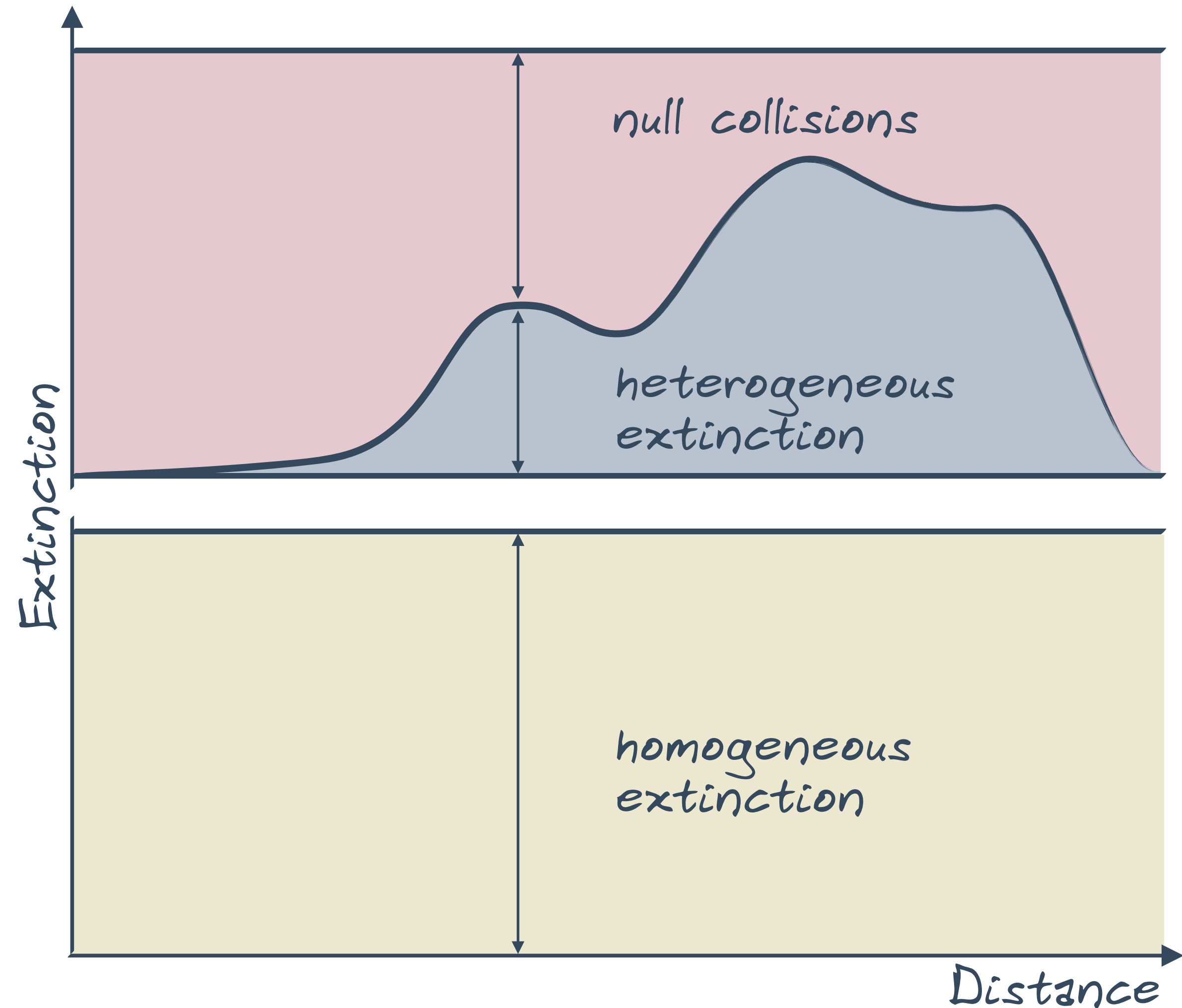
DECOMPOSITION TRACKING



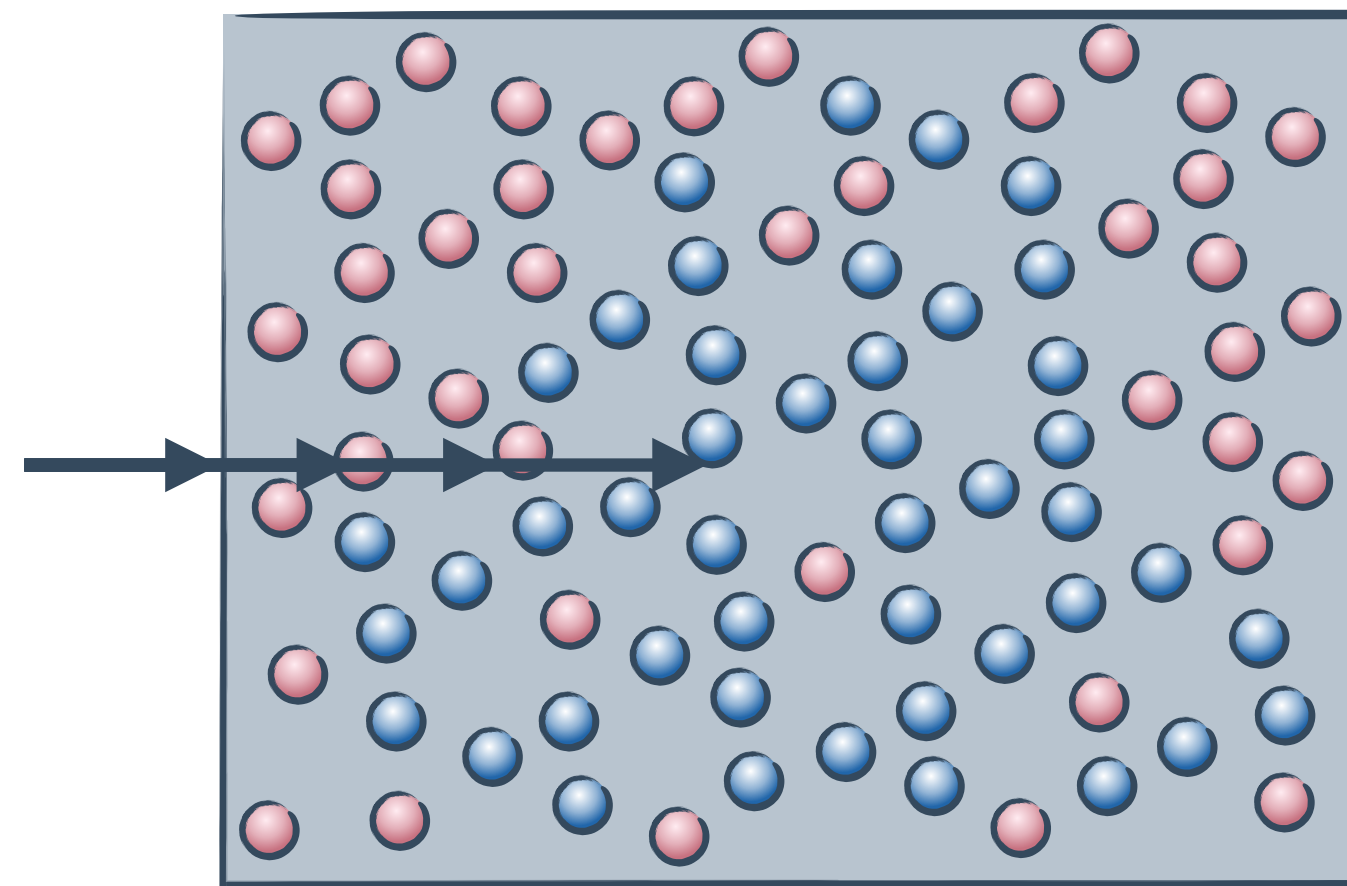
HETEROGENEOUS
component



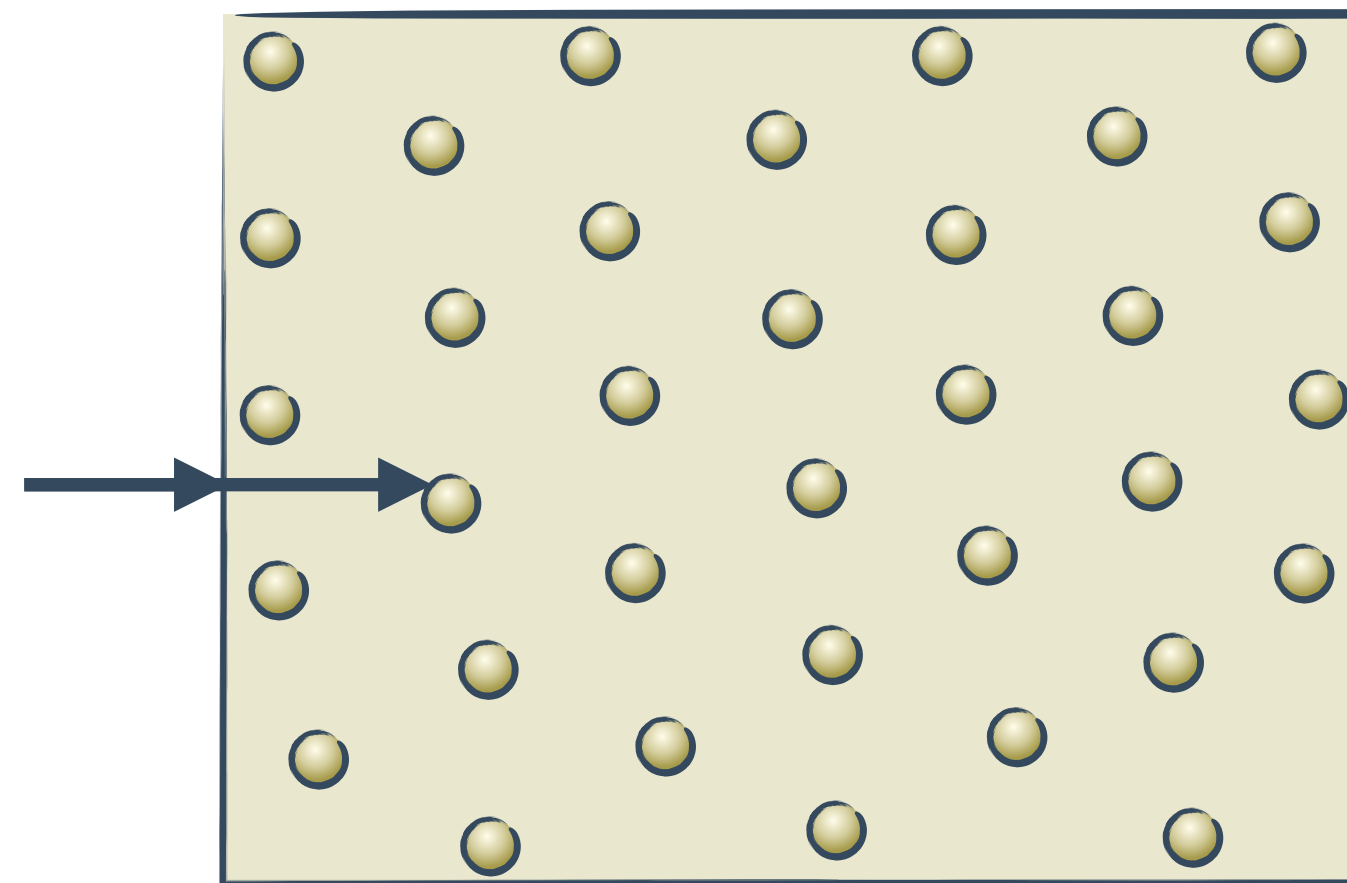
HOMOGENEOUS
component



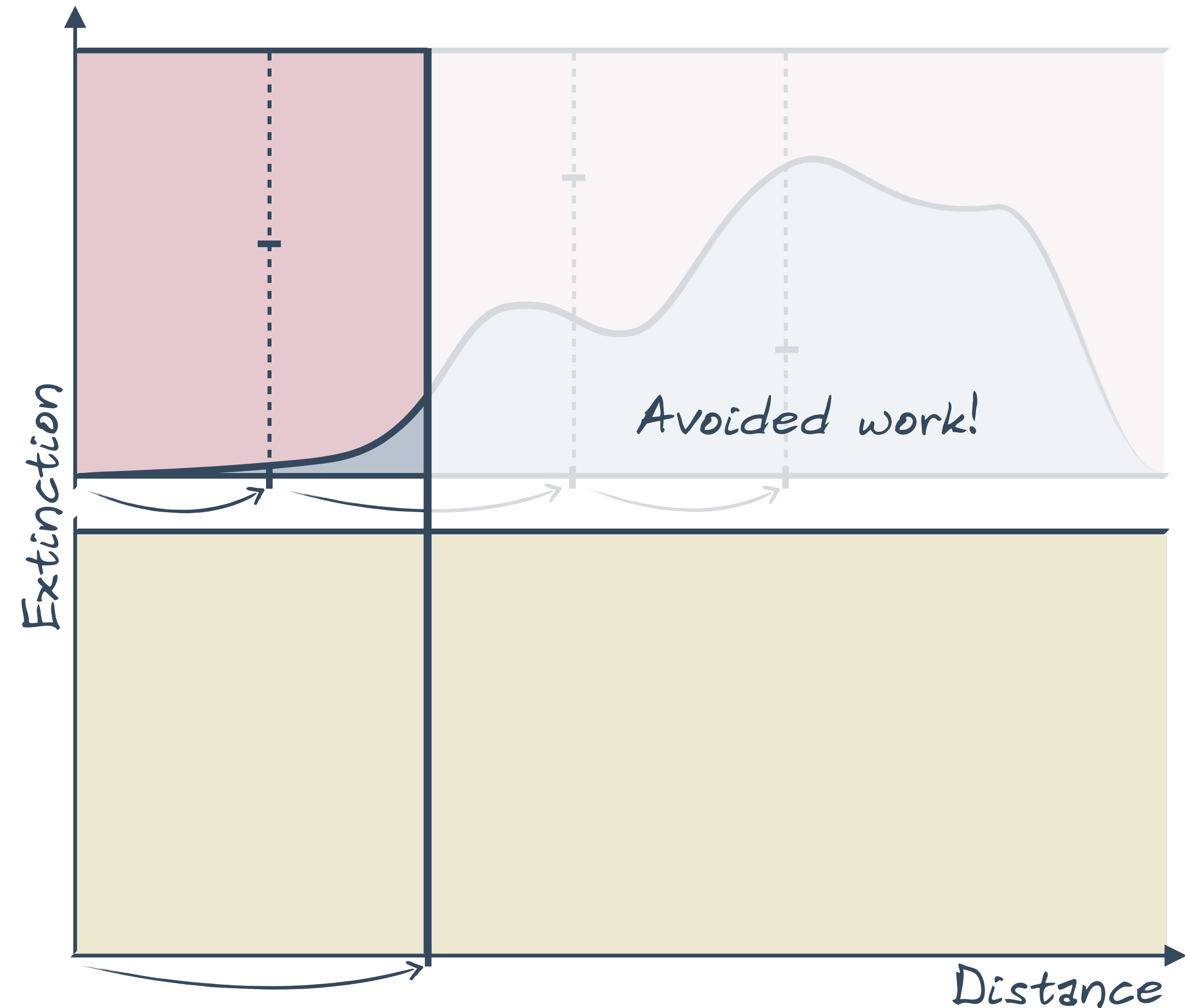
DECOMPOSITION TRACKING



HETEROGENEOUS
component



HOMOGENEOUS
component



DECOMPOSITION TRACKING

Decomposition tracking:

1) Decompose into homogeneous and heterogeneous

2) Sample homogeneous component

Repeat

3) Sample tentative free path in heterogeneous component

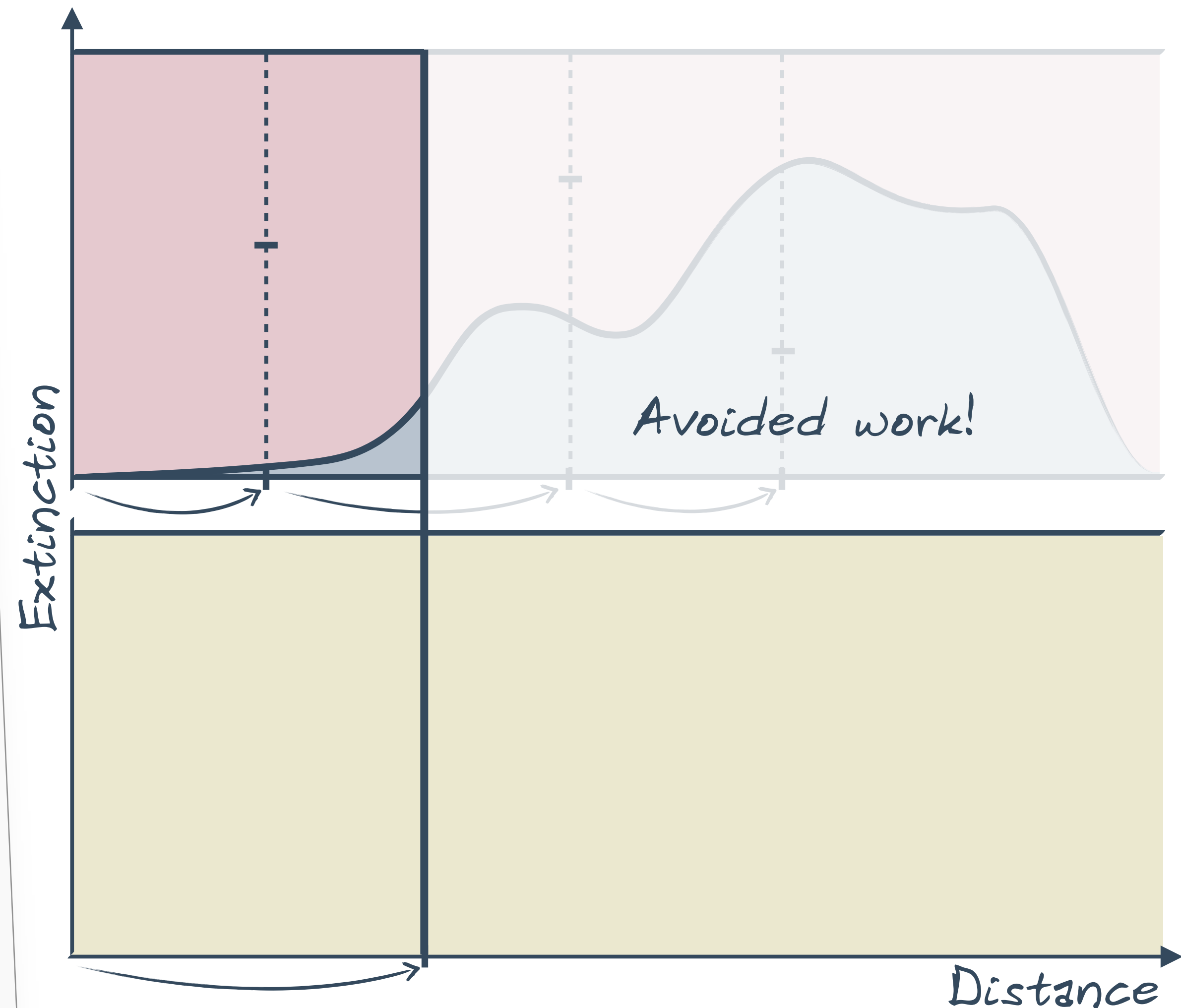
4) If beyond homogeneous sample

5) Return homogeneous sample

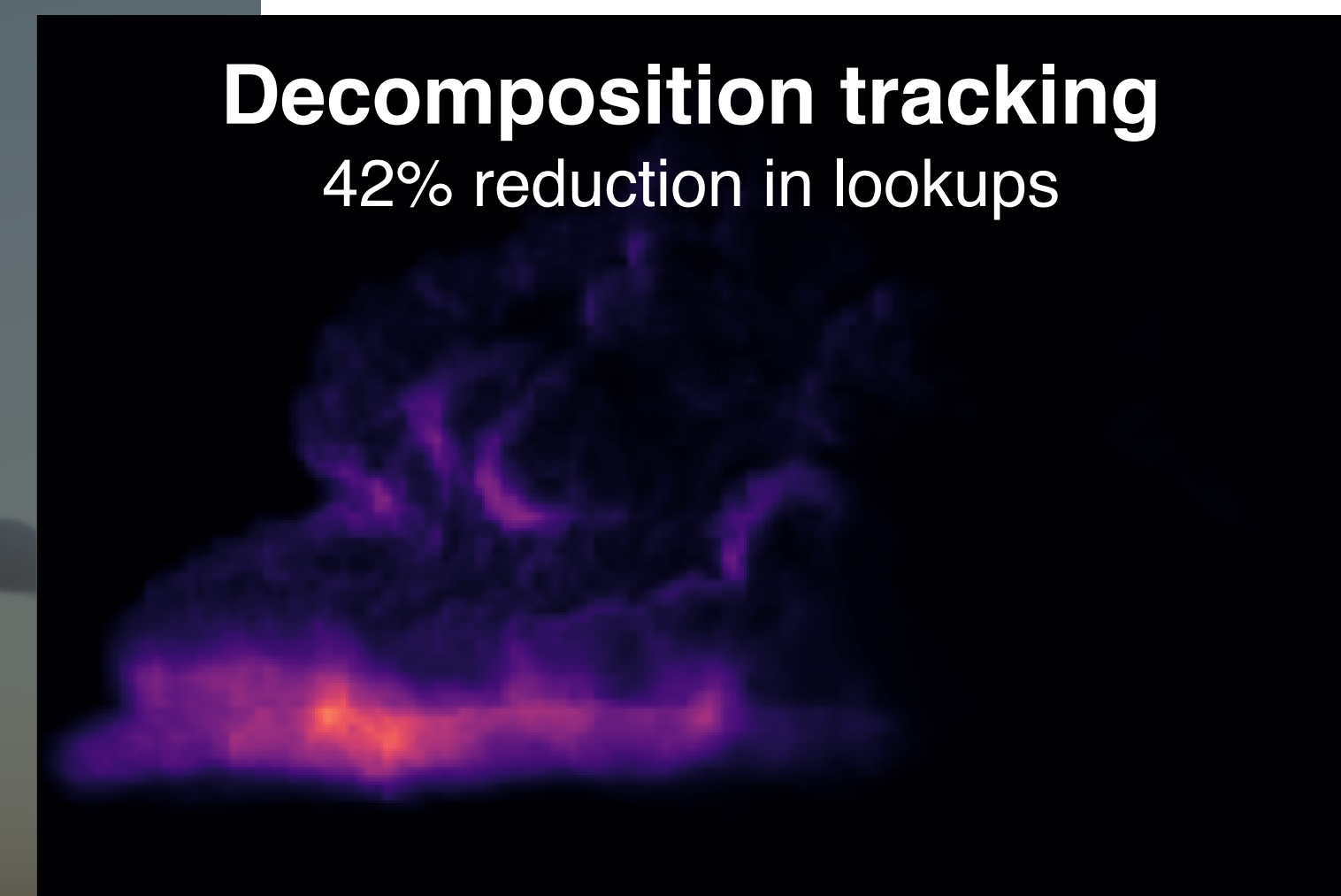
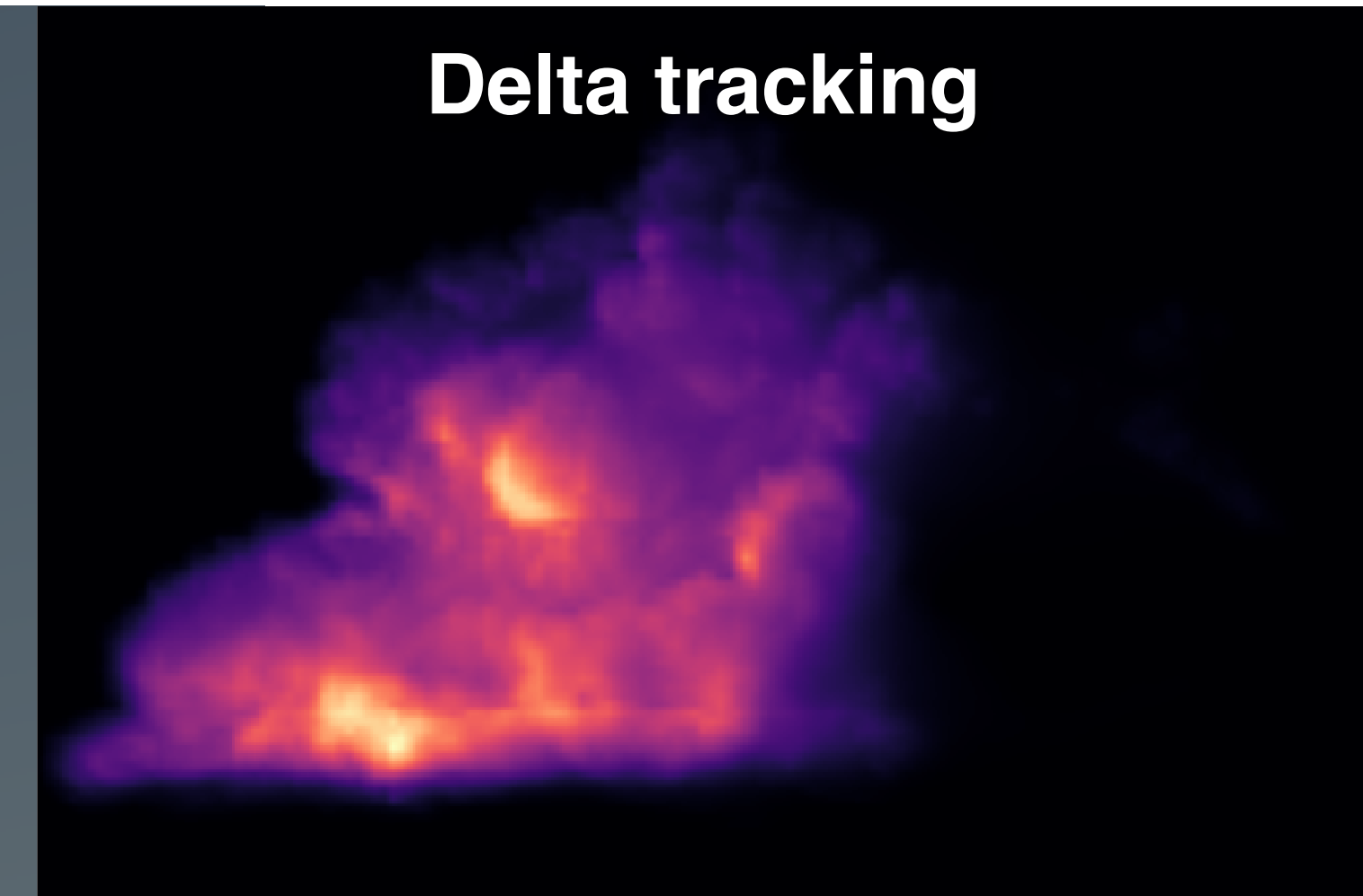
6) Probabilistically classify collision

Until collision classified as real

7) Return heterogeneous sample



DECOMPOSITION TRACKING



Kutz et al. [2017]

HOMOGENEOUS and RESIDUAL HETEROGENEOUS components

- ▶ Reduces evaluations of spatially varying collision coefficients
- ▶ Requires a space-partitioning data structure (e.g. octree) to be practical
- ▶ Can be combine with weighted tracking to handle arbitrary decompositions

MORE DISTANCE SAMPLING...

- ▶ Equiangular sampling
[Kulla and Fajardo 2012]
 - ▶ Joint-importance sampling
[Georgiev et al. 2013]
 - ▶ Tabulation approaches
[Kulla and Fajardo 2012, Novák et al. 2012, Georgiev et al. 2013, Novák et al. 2014]
- Discussed by
Iliyan later*

...END OF THIS PART

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