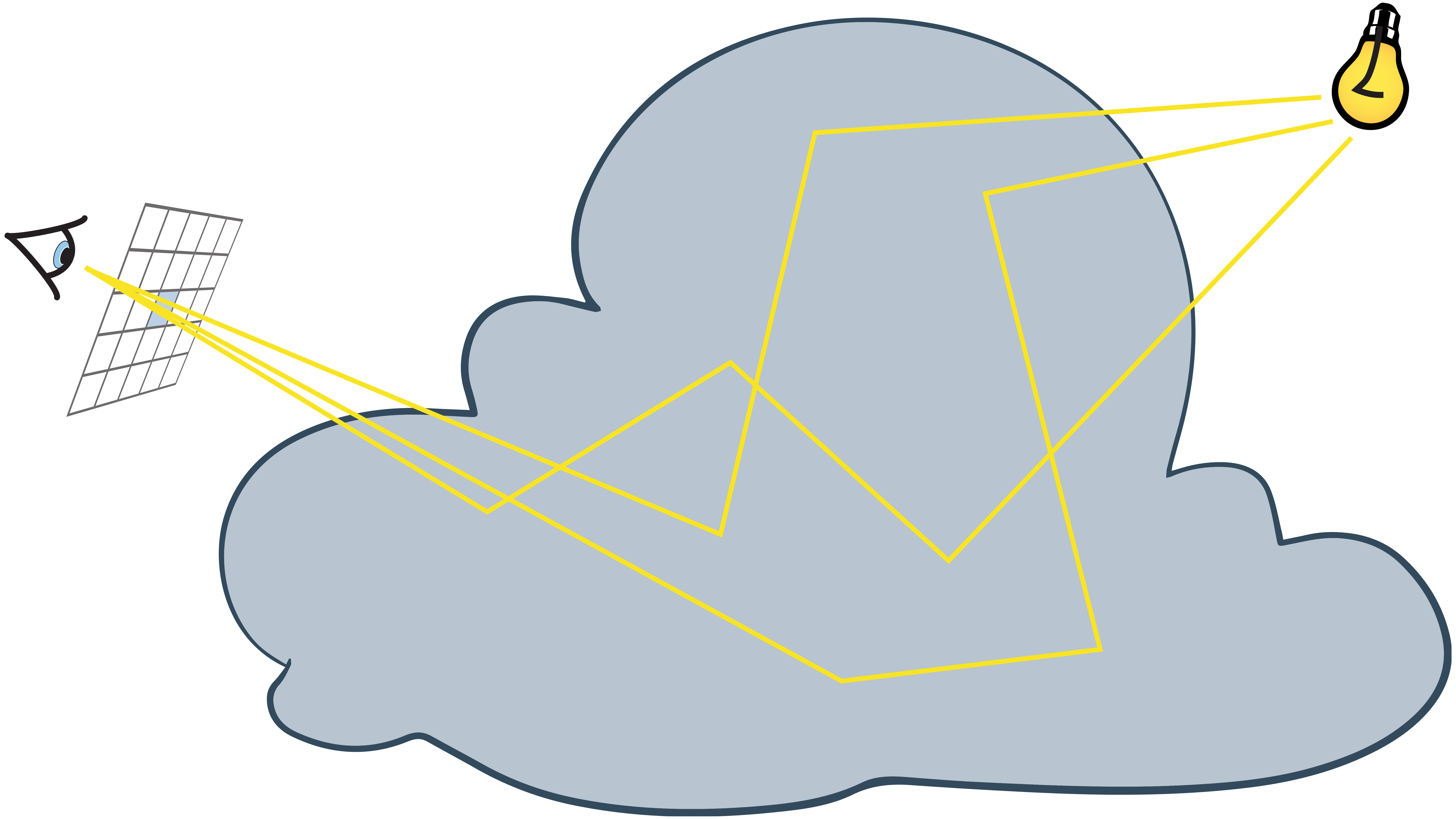
PATH CONSTRUCTION

Iliyan Georgiev
Solid Angle

PATH INTEGRAL FRAMEWORK



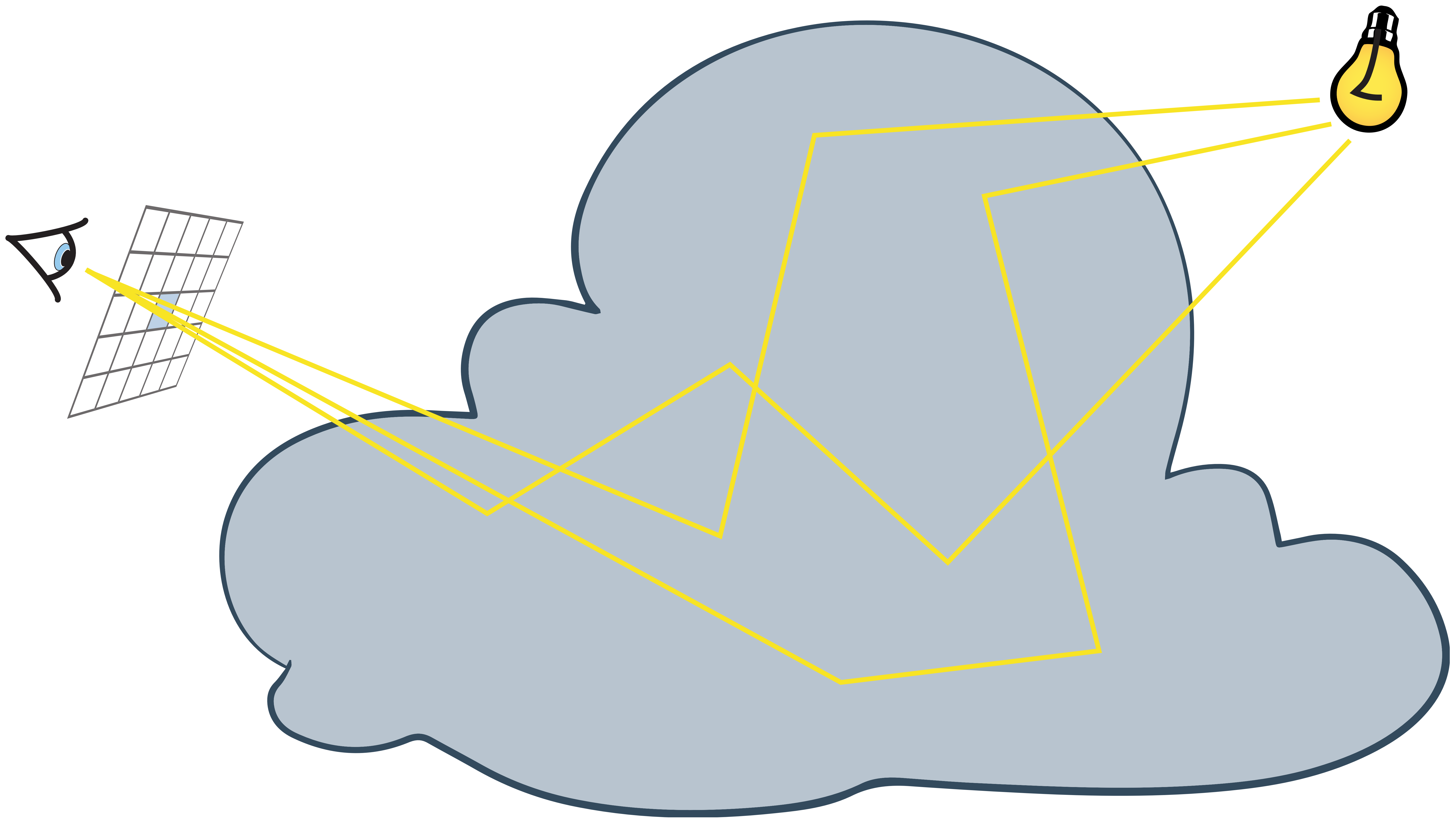
Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\overline{\mathbf{x}}) d\overline{\mathbf{x}}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\overline{\mathbf{x}}_i)}{p(\overline{\mathbf{x}}_i)}$$

PATH INTEGRAL FRAMEWORK



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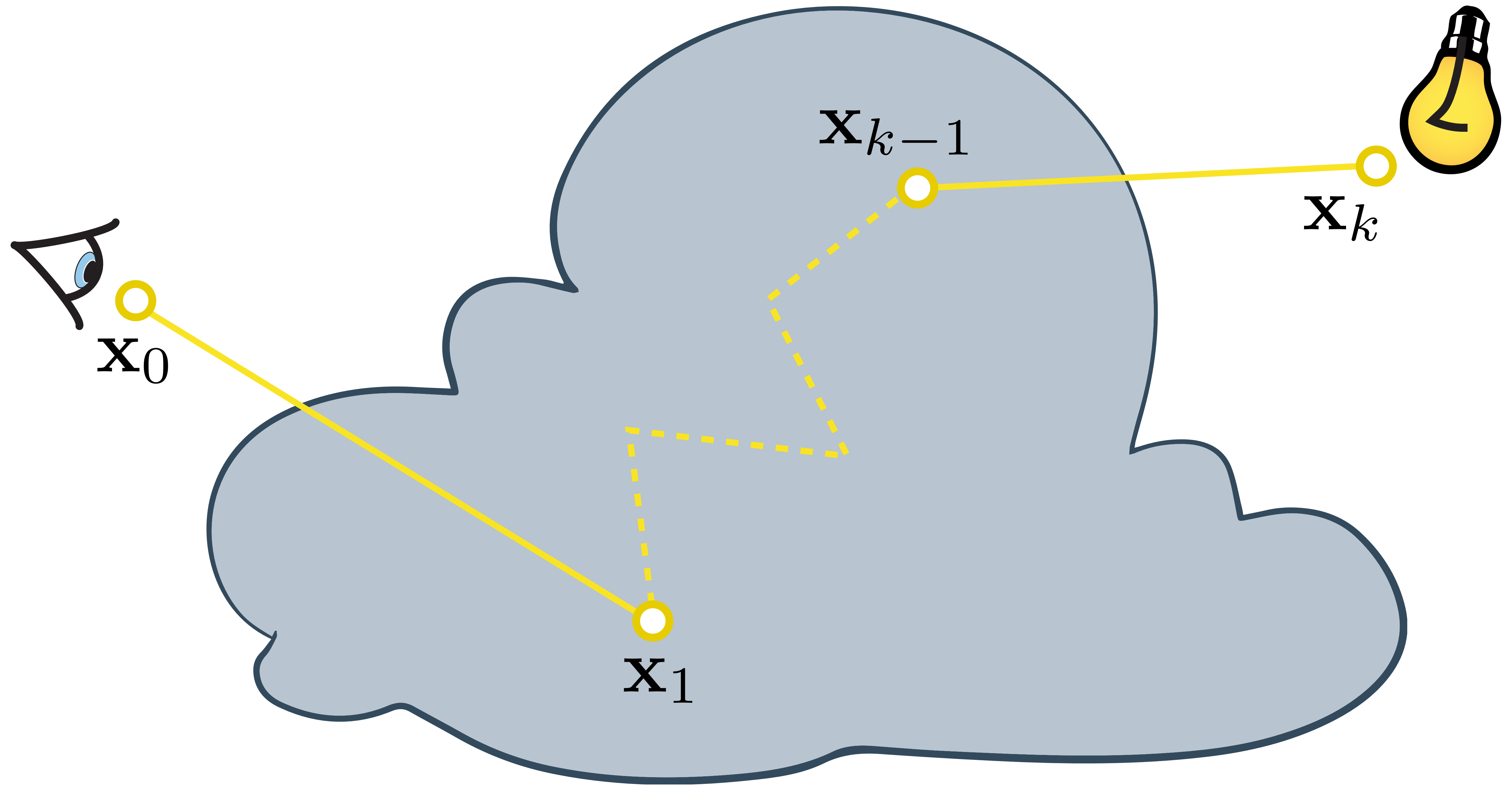
path contribution

path pdf

PATH INTEGRAL FRAMEWORK

EUROGRAPHICS 2018



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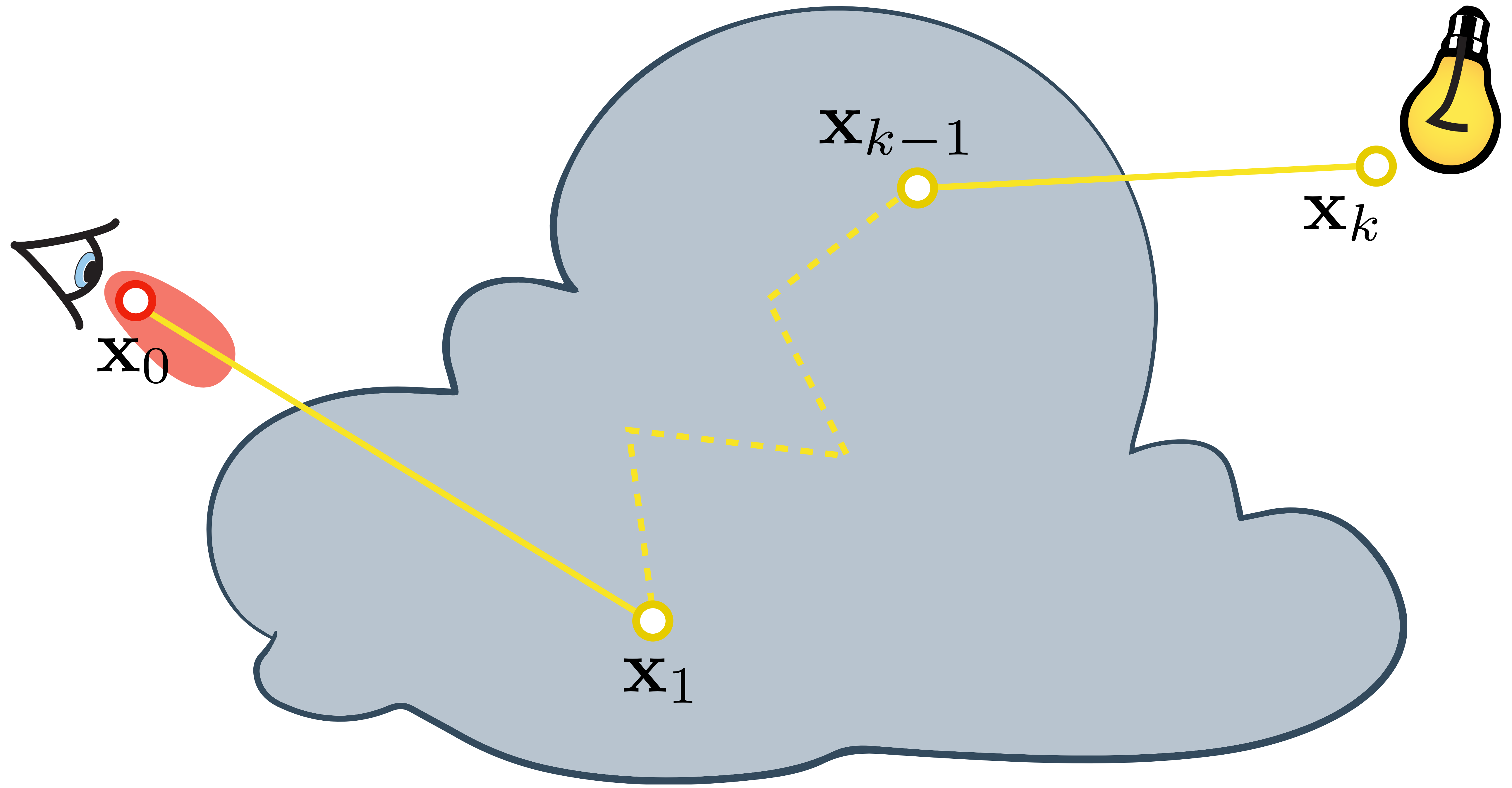
Path contribution

$$f_j(\overline{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[\prod_i f_{\mathrm{s}}(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_{\mathrm{e}}(\mathbf{x}_k, \mathbf{x}_{k-1})$$

PATH INTEGRAL FRAMEWORK



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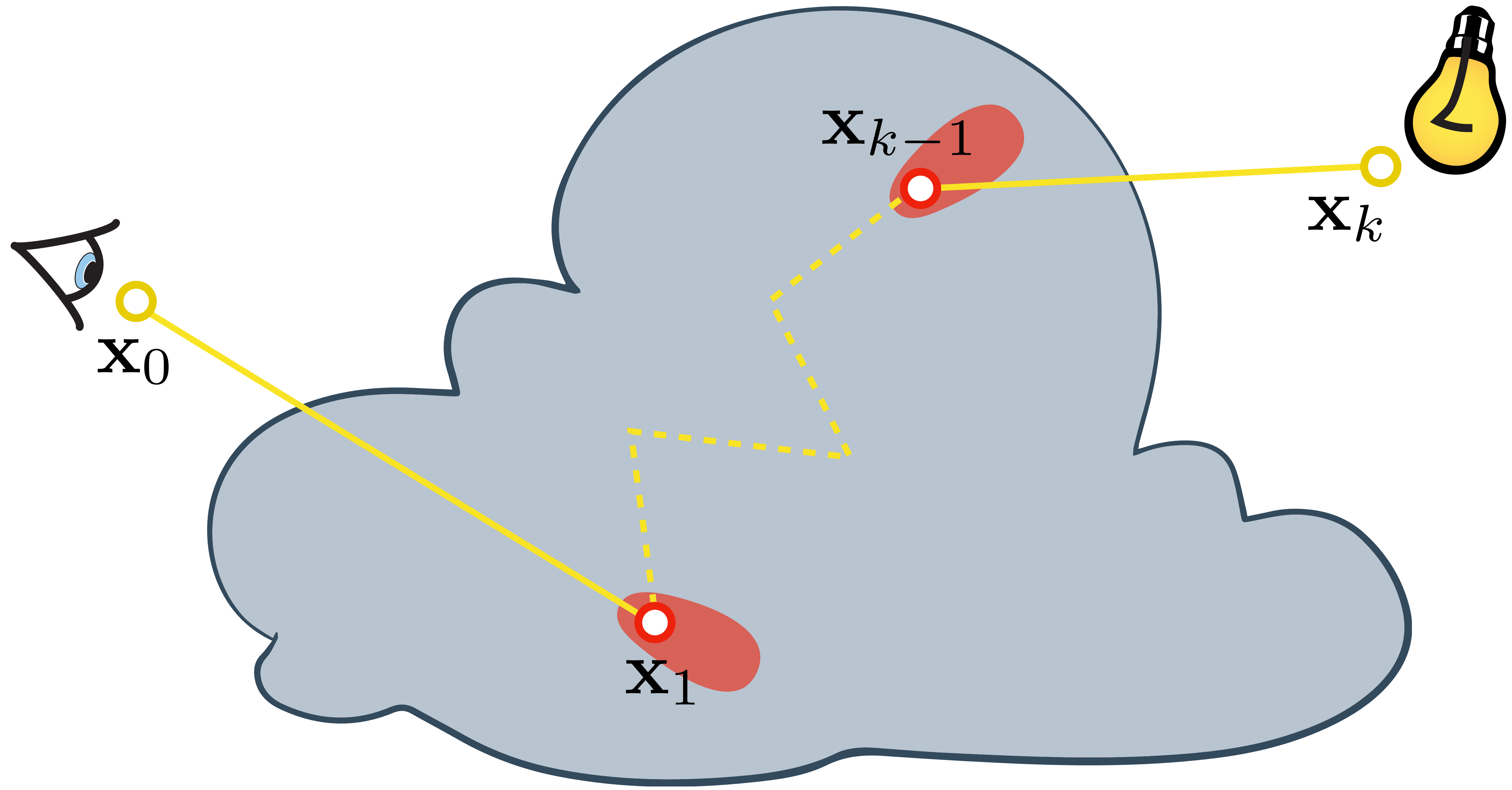
camera response

PATH INTEGRAL FRAMEWORK



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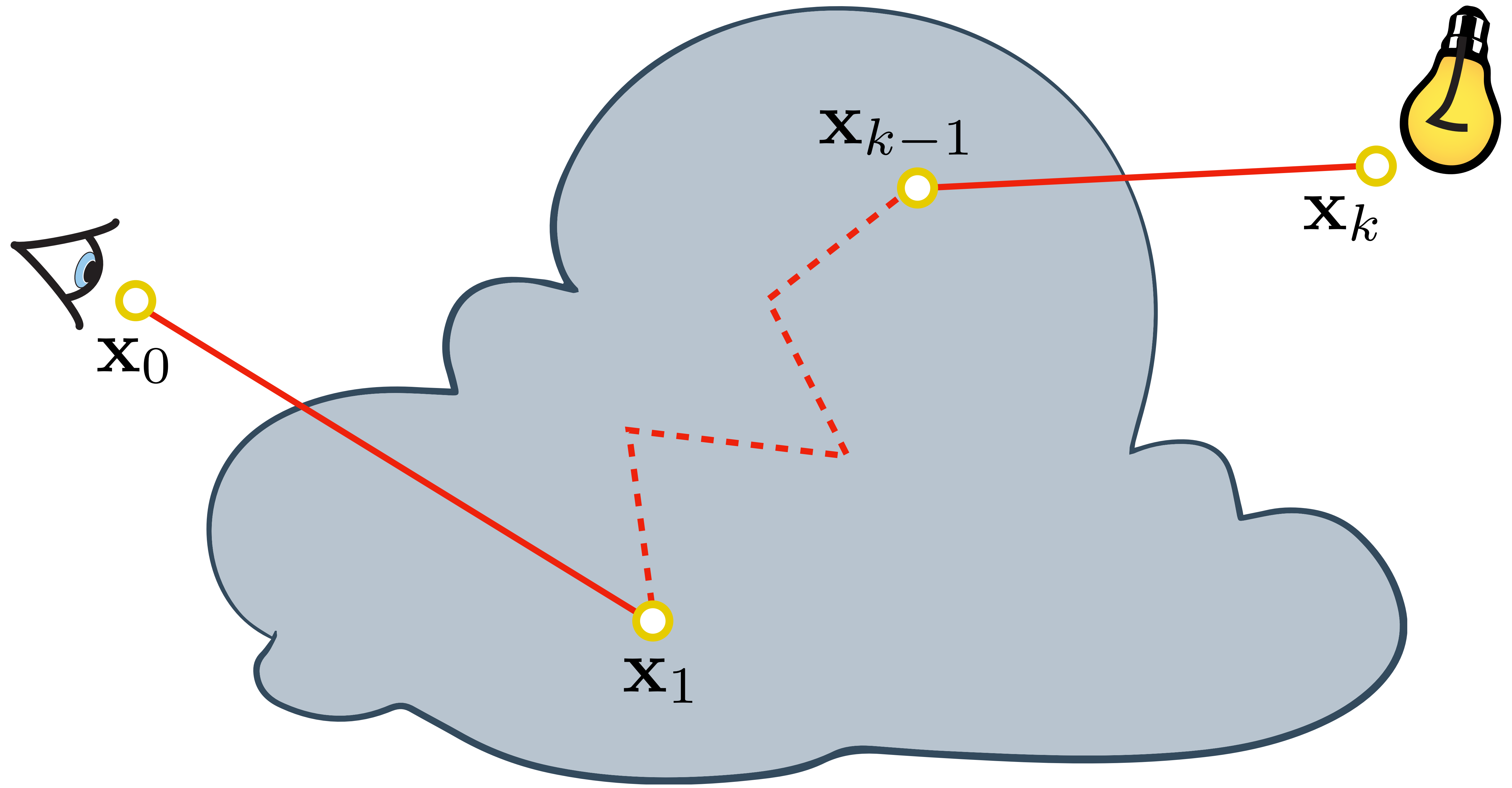
camera response

BSDF/ phase

PATH INTEGRAL FRAMEWORK

EUROGRAPHICS 2018



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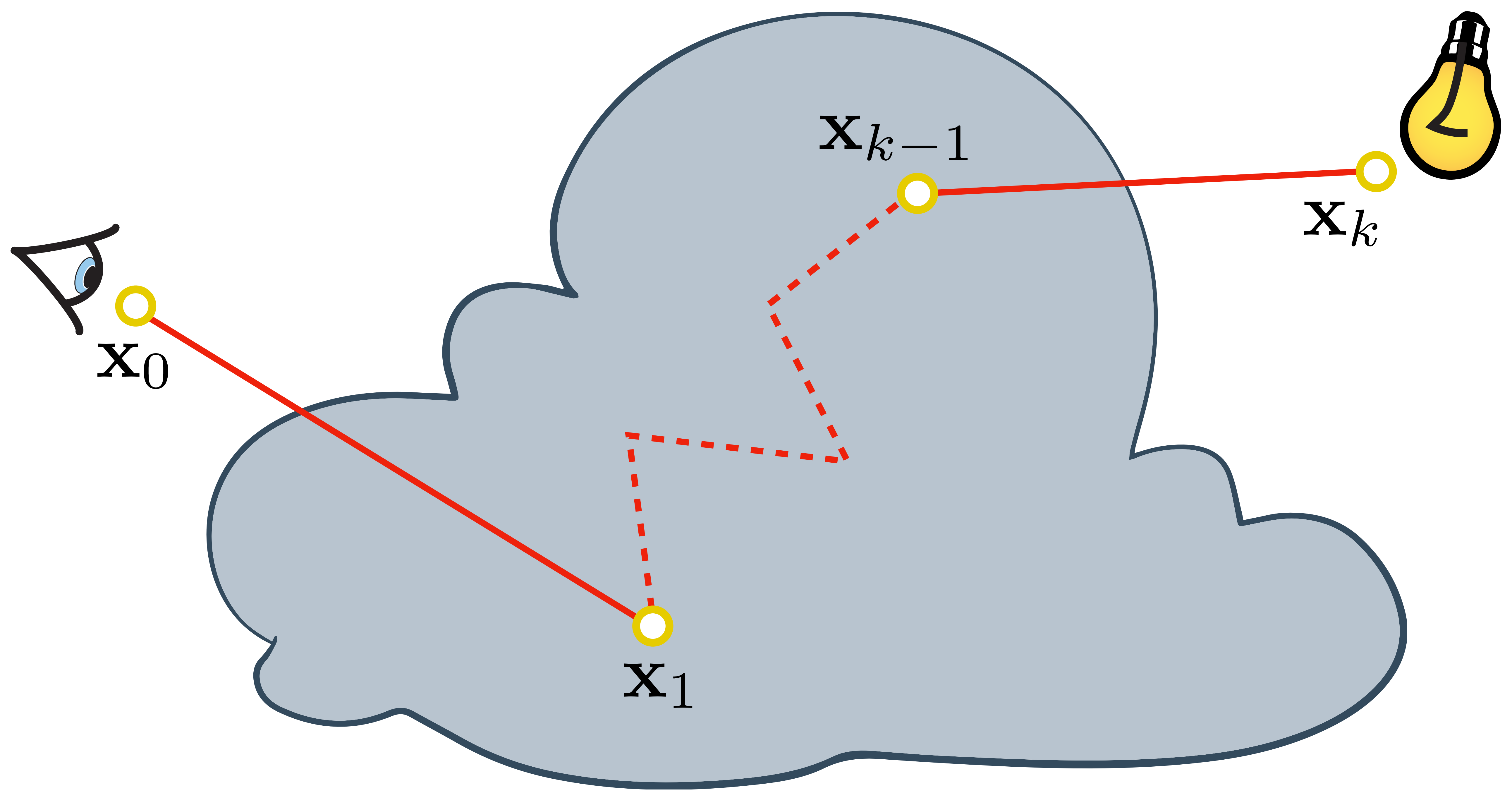
camera response

BSDF/ phase

geometry

PATH INTEGRAL FRAMEWORK



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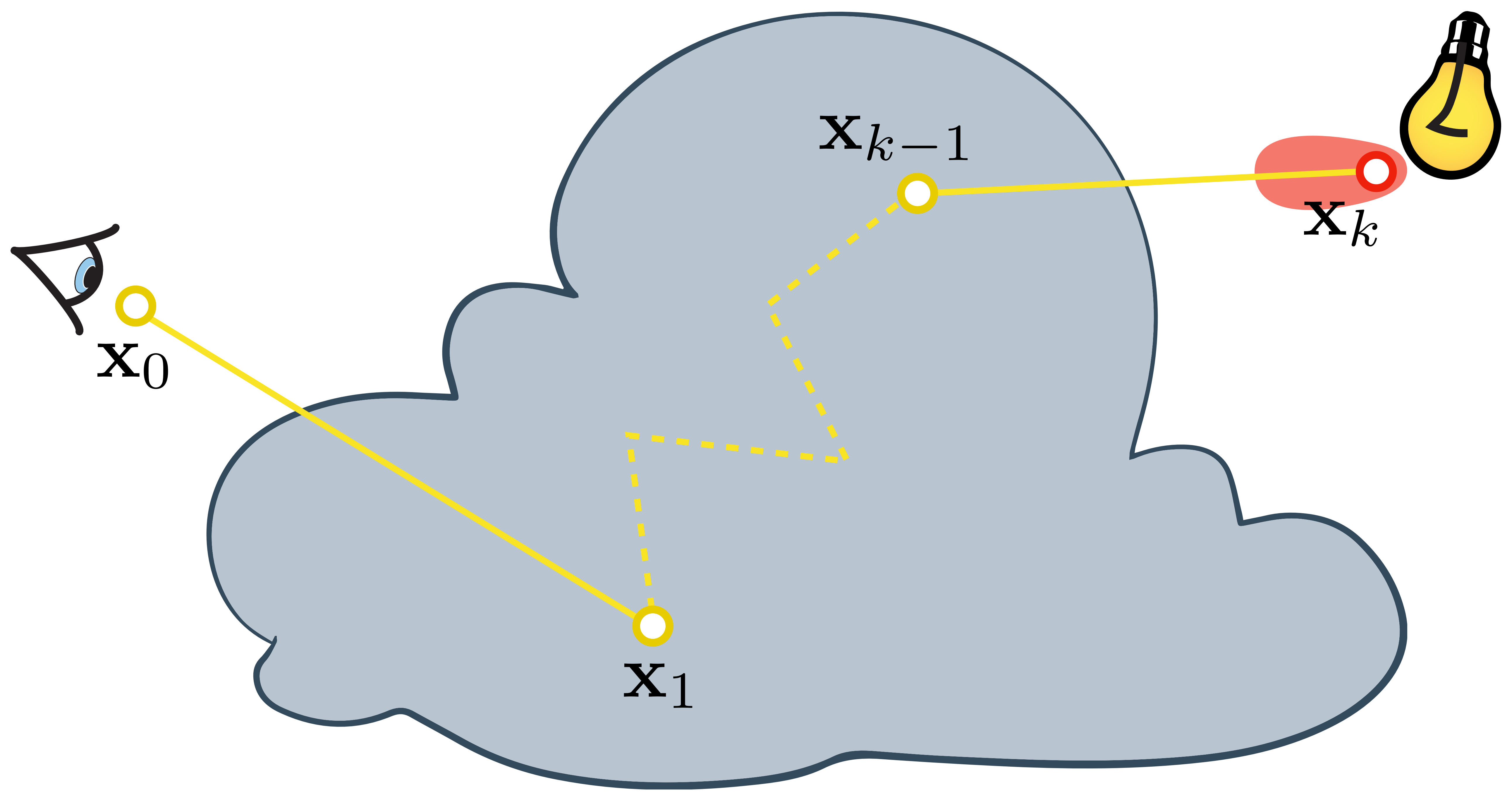
camera response — BSDF/ phase — geometry — transmittance

PATH INTEGRAL FRAMEWORK



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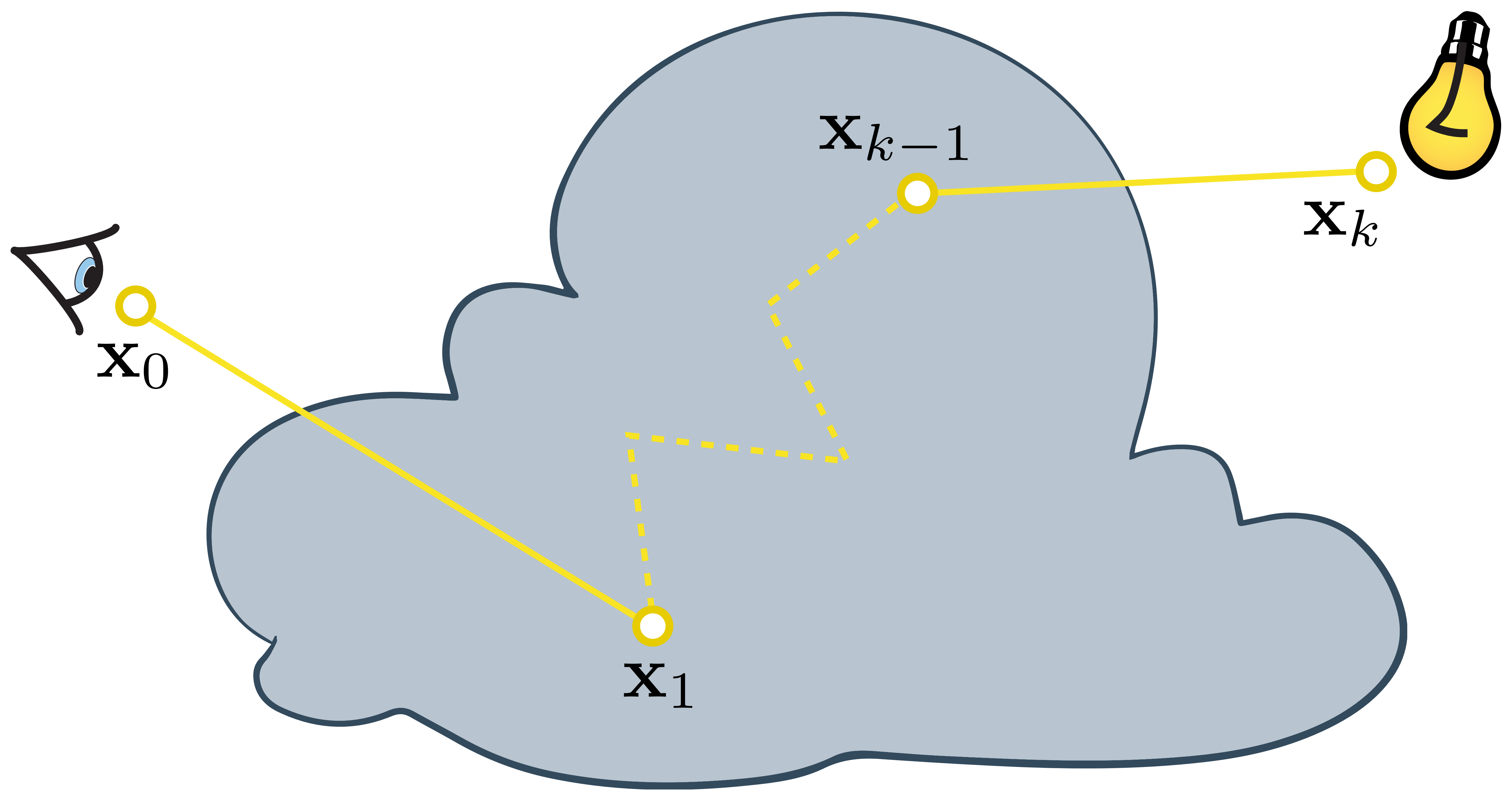
camera response; BSDF/ phase; geometry; transmittance; emitted radiance

PATH INTEGRAL FRAMEWORK



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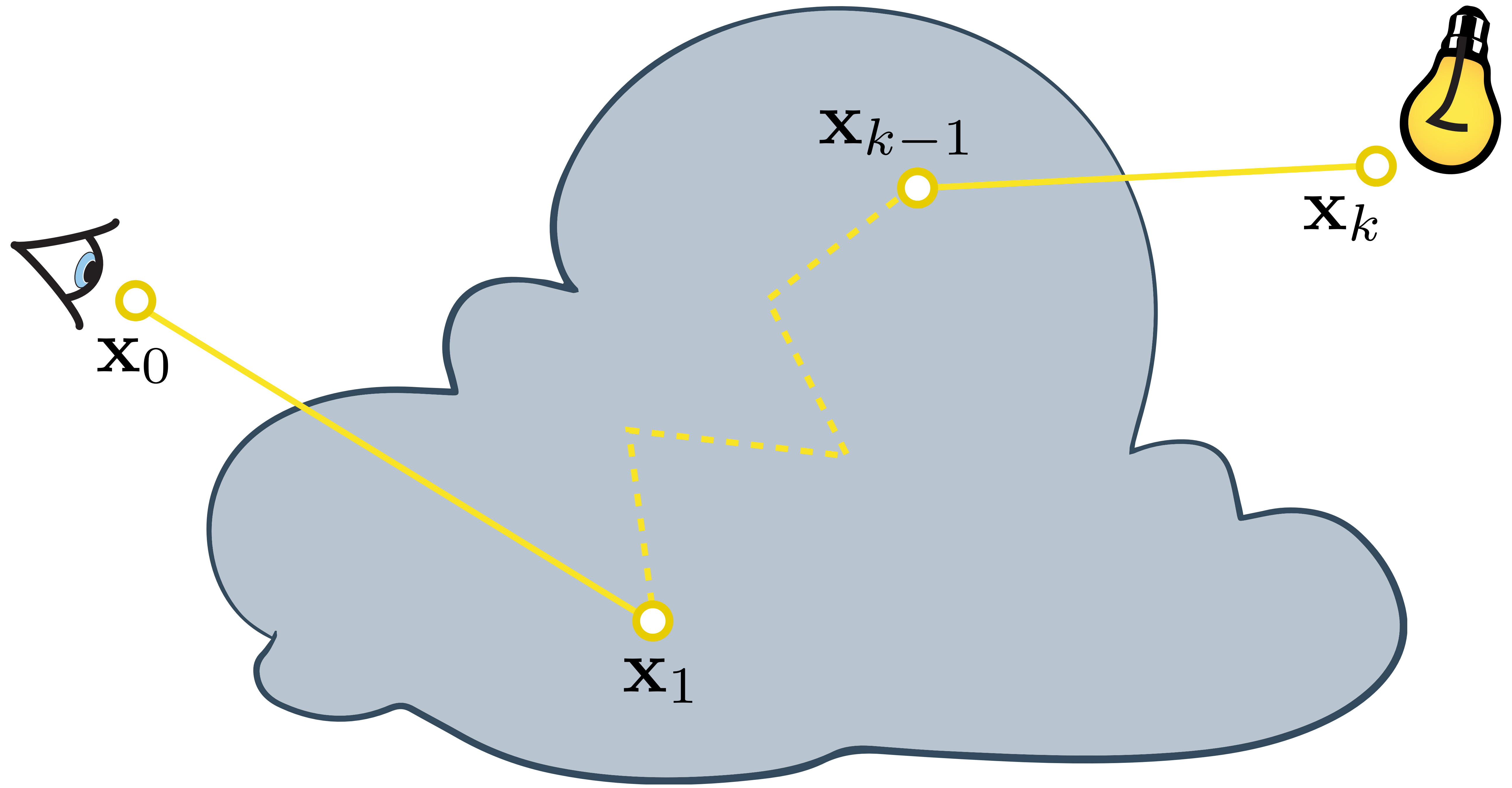
camera response · BSDF/phase · geometry · transmittance · emitted radiance

PATH INTEGRAL FRAMEWORK



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Pixel estimator

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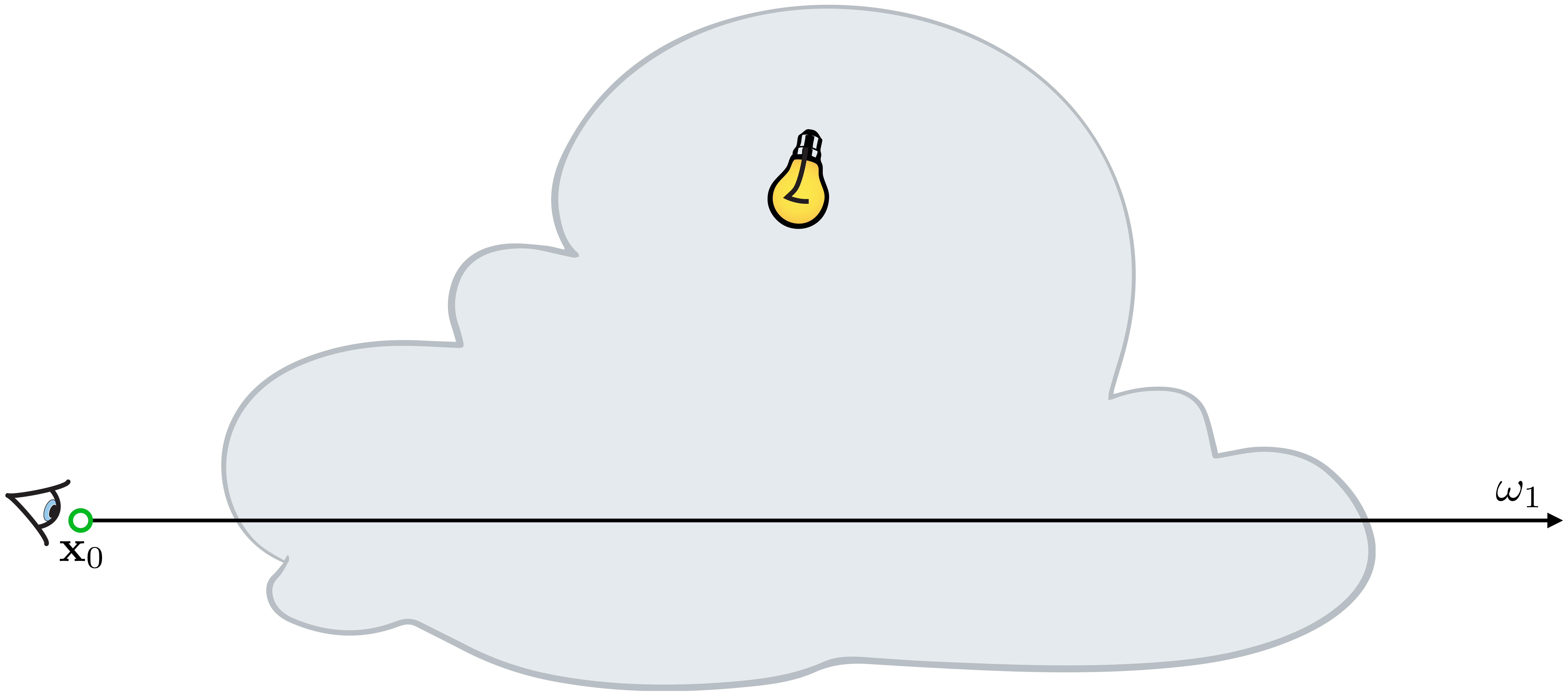
ideally proportional $\propto$

Path contribution

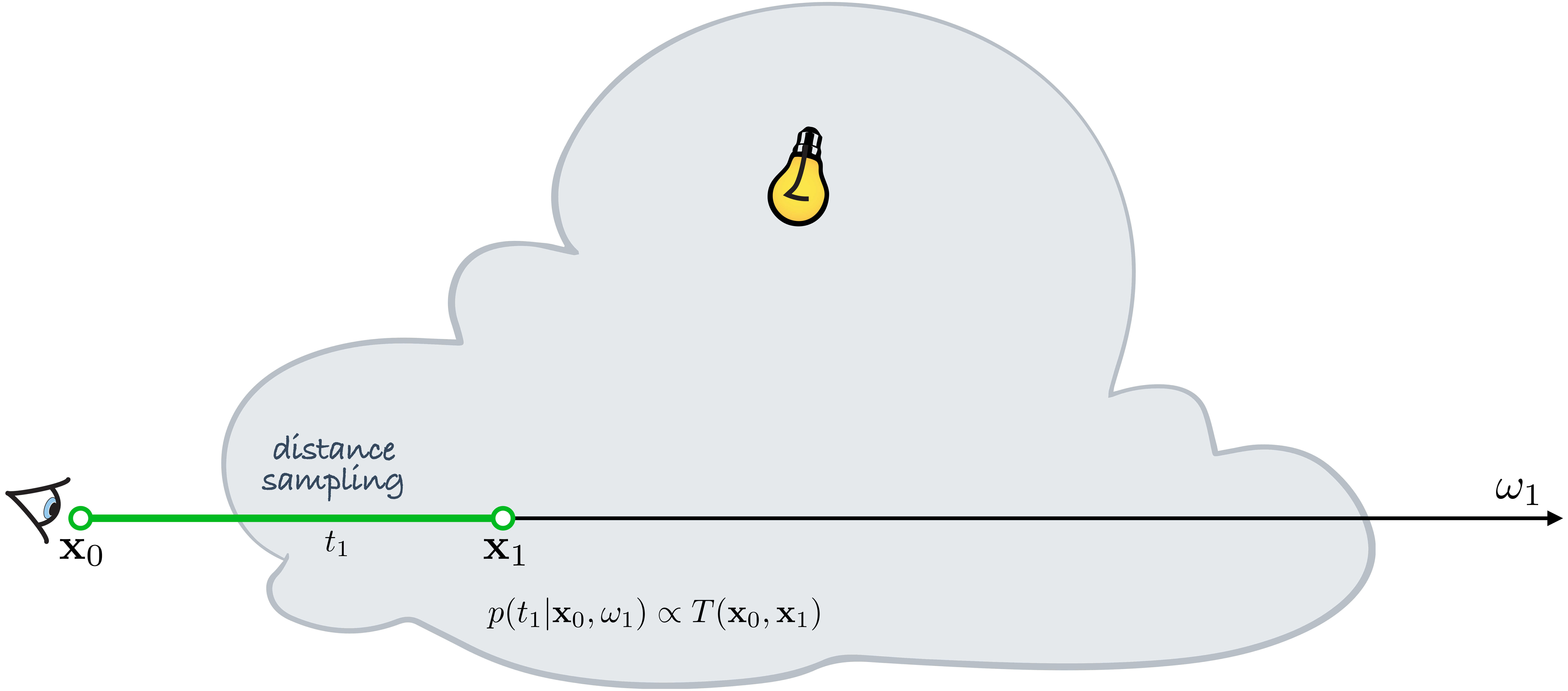
$$f_j(\overline{\mathbf{x}}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[\prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

camera response | BSDF/ phase | geometry | transmittance | emitted radiance

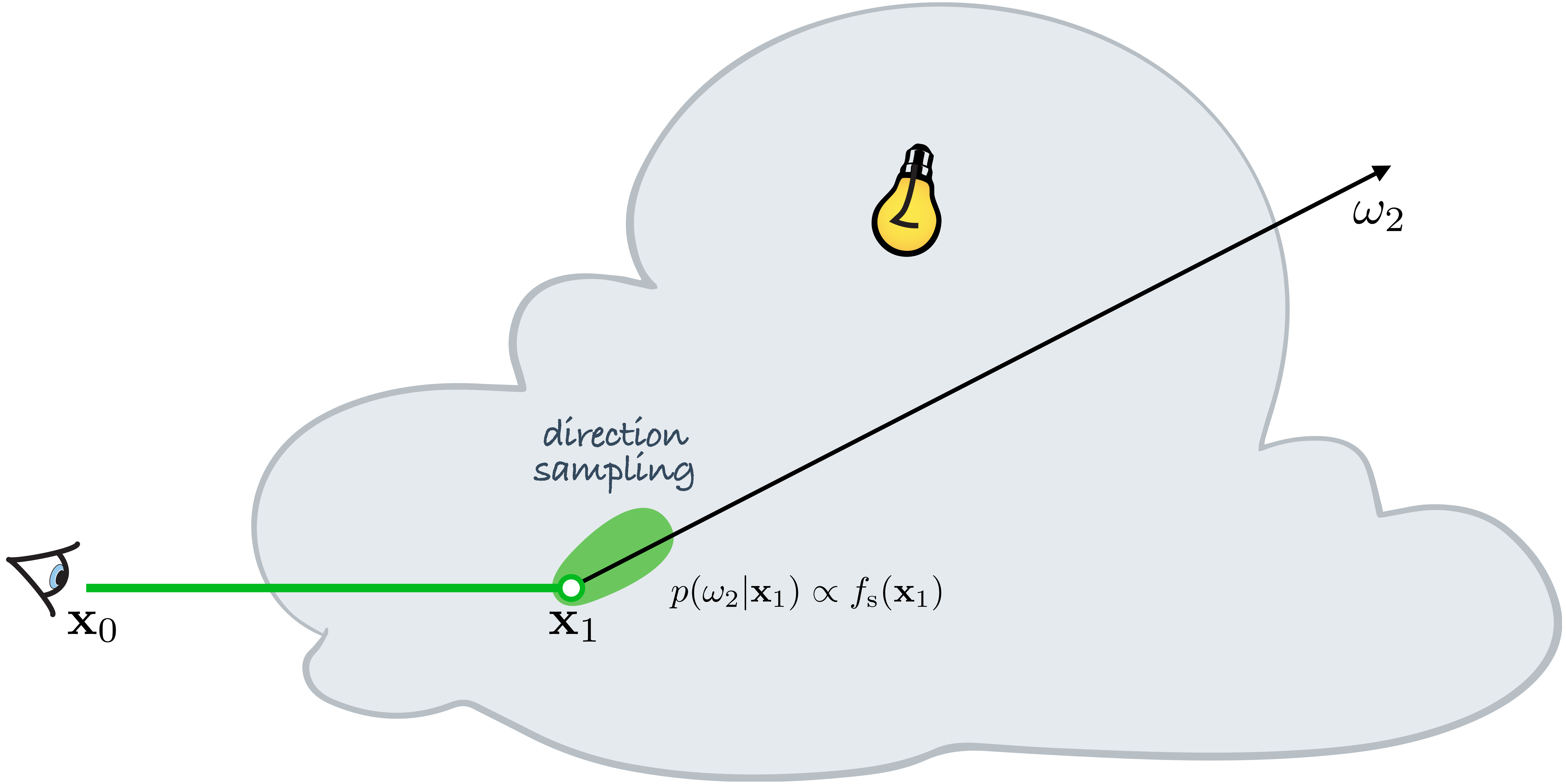
UNIDIRECTIONAL PATH SAMPLING



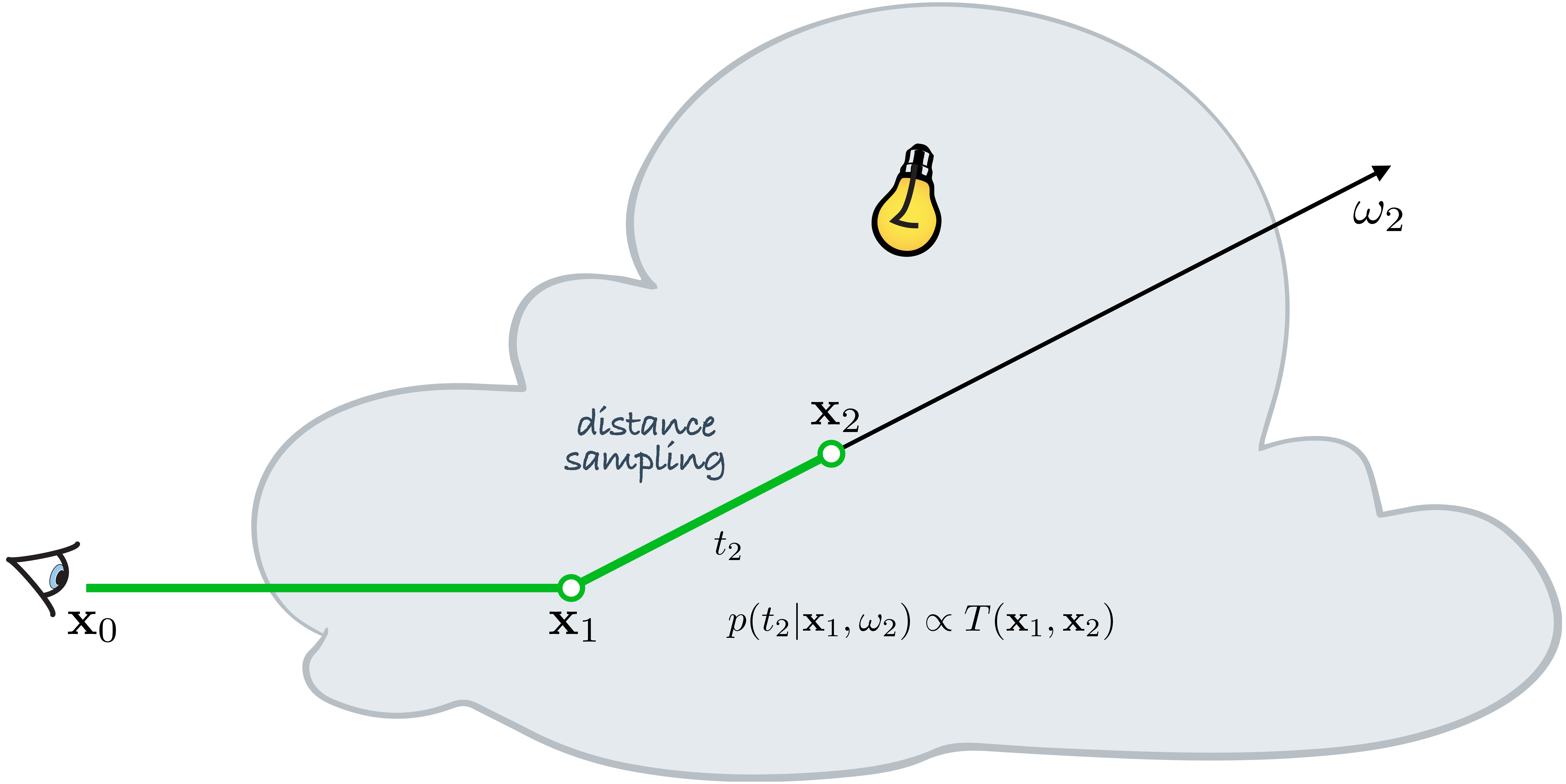
UNIDIRECTIONAL PATH SAMPLING



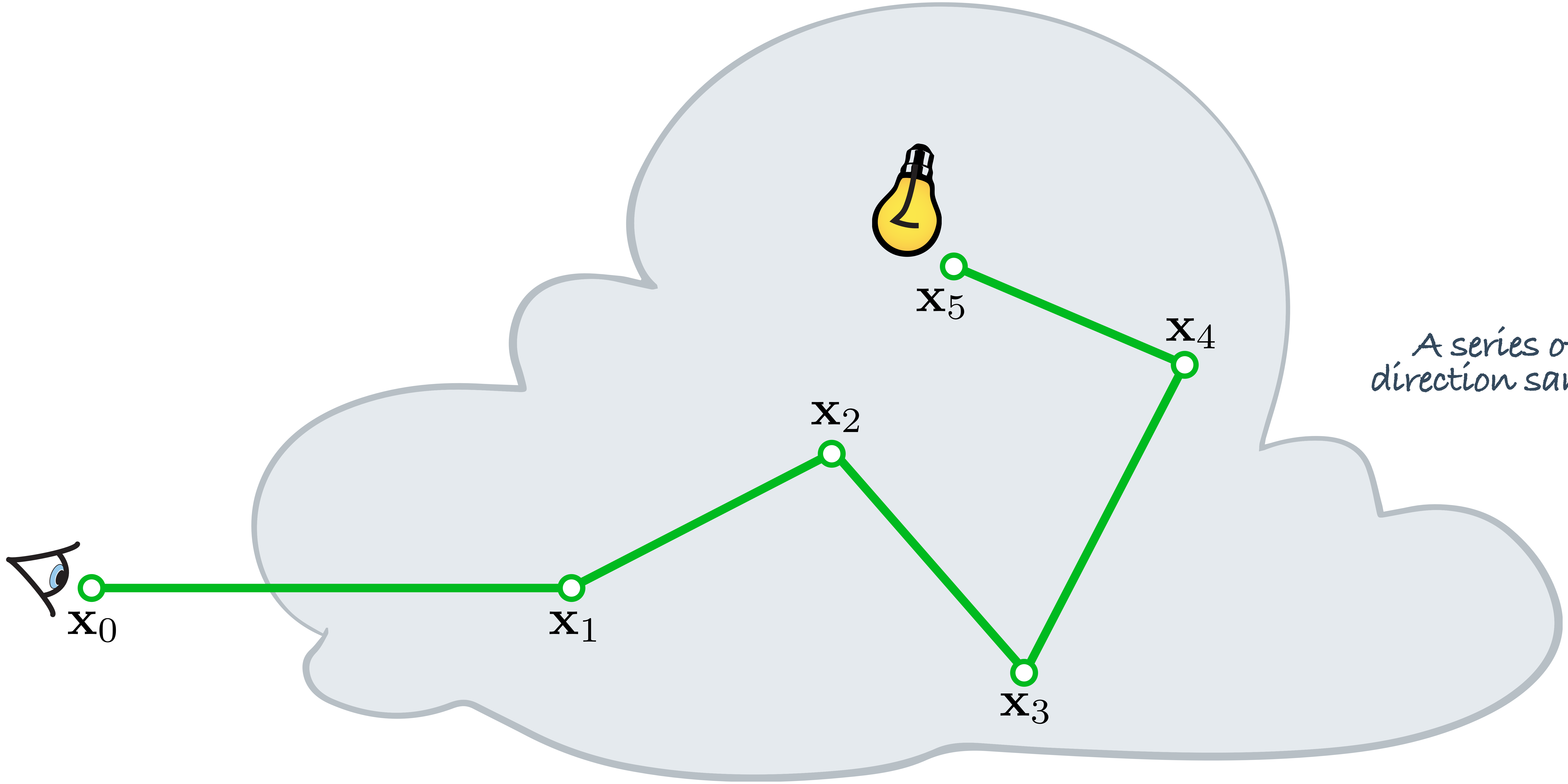
UNIDIRECTIONAL PATH SAMPLING



UNIDIRECTIONAL PATH SAMPLING



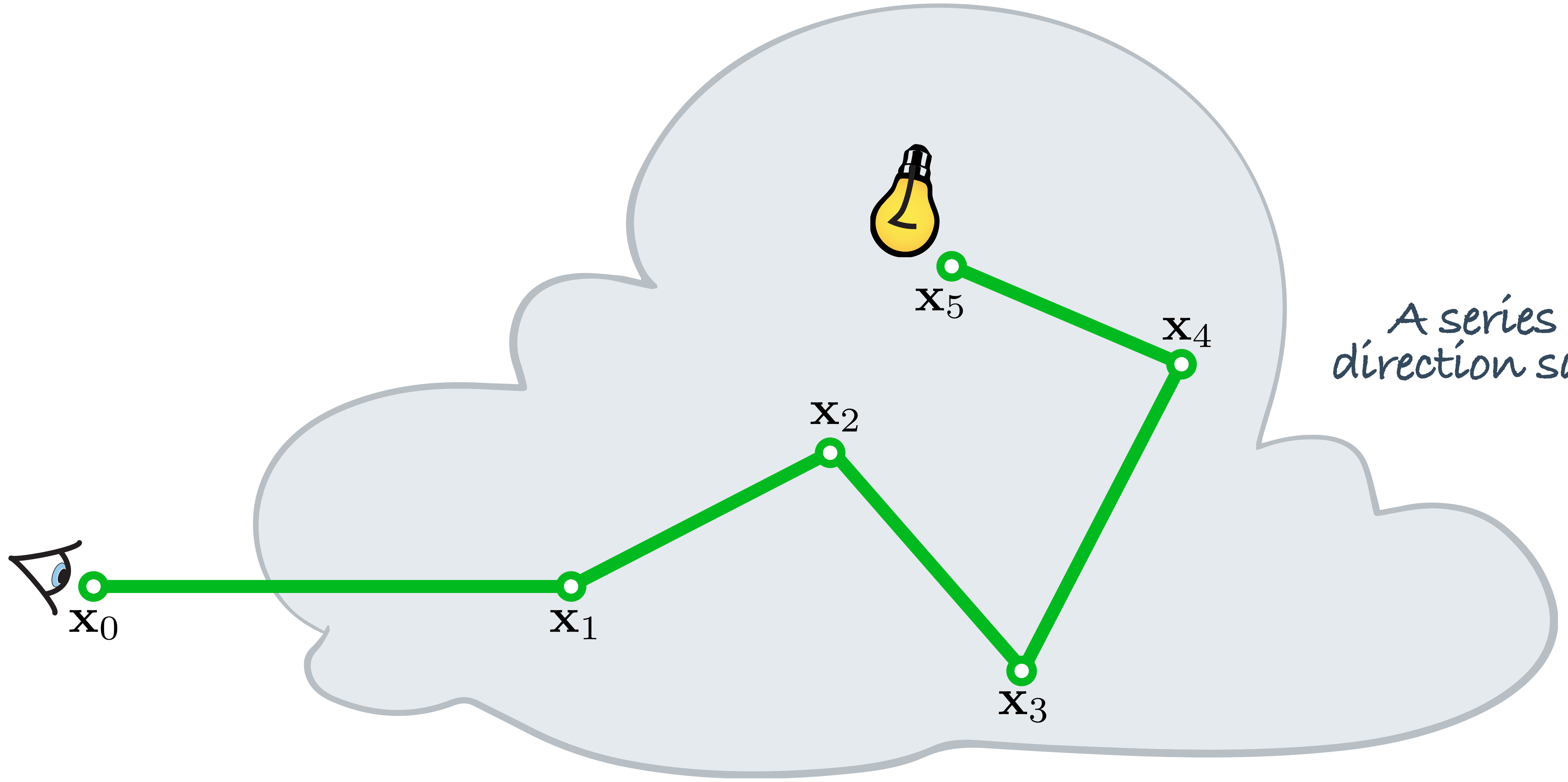
UNIDIRECTIONAL PATH SAMPLING



A series of distance and direction sampling decisions

UNIDIRECTIONAL PATH SAMPLING

EUROGRAPHICS 2018



A series of distance and direction sampling decisions

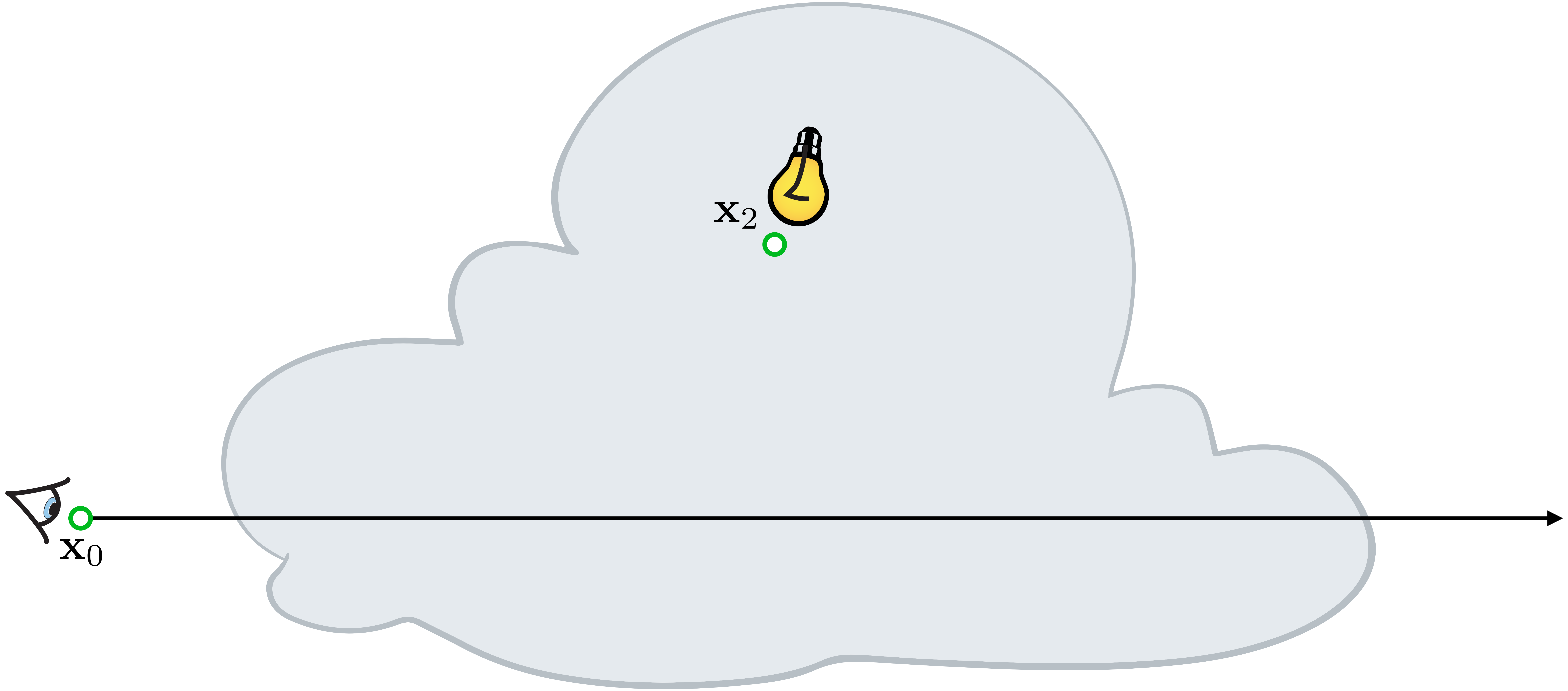
cannot render illumination from point light sources

high variance when light sources are small

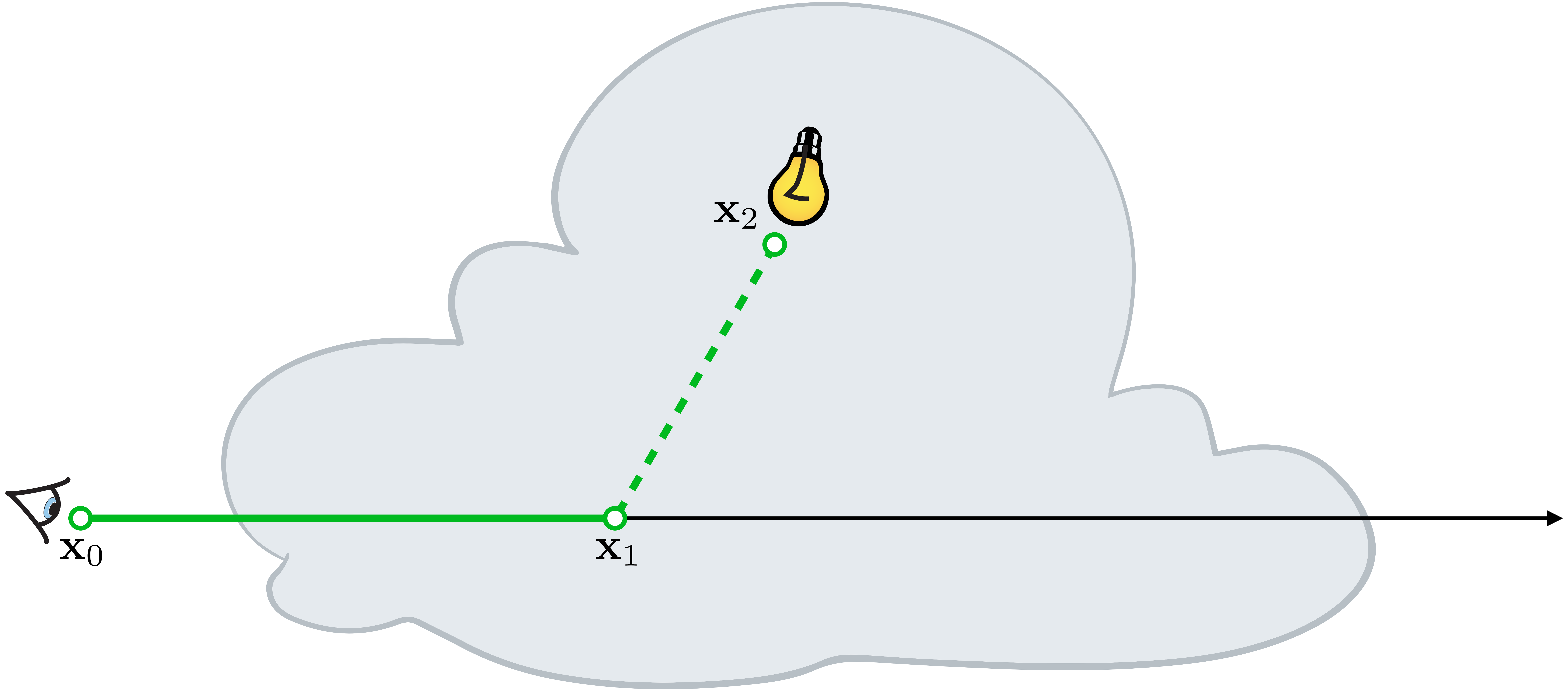
not importance sampled

$$p(\overline{\mathbf{x}}) \propto W_j(\mathbf{x}_0, \mathbf{x}_1) \left[\prod_i f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1})$$

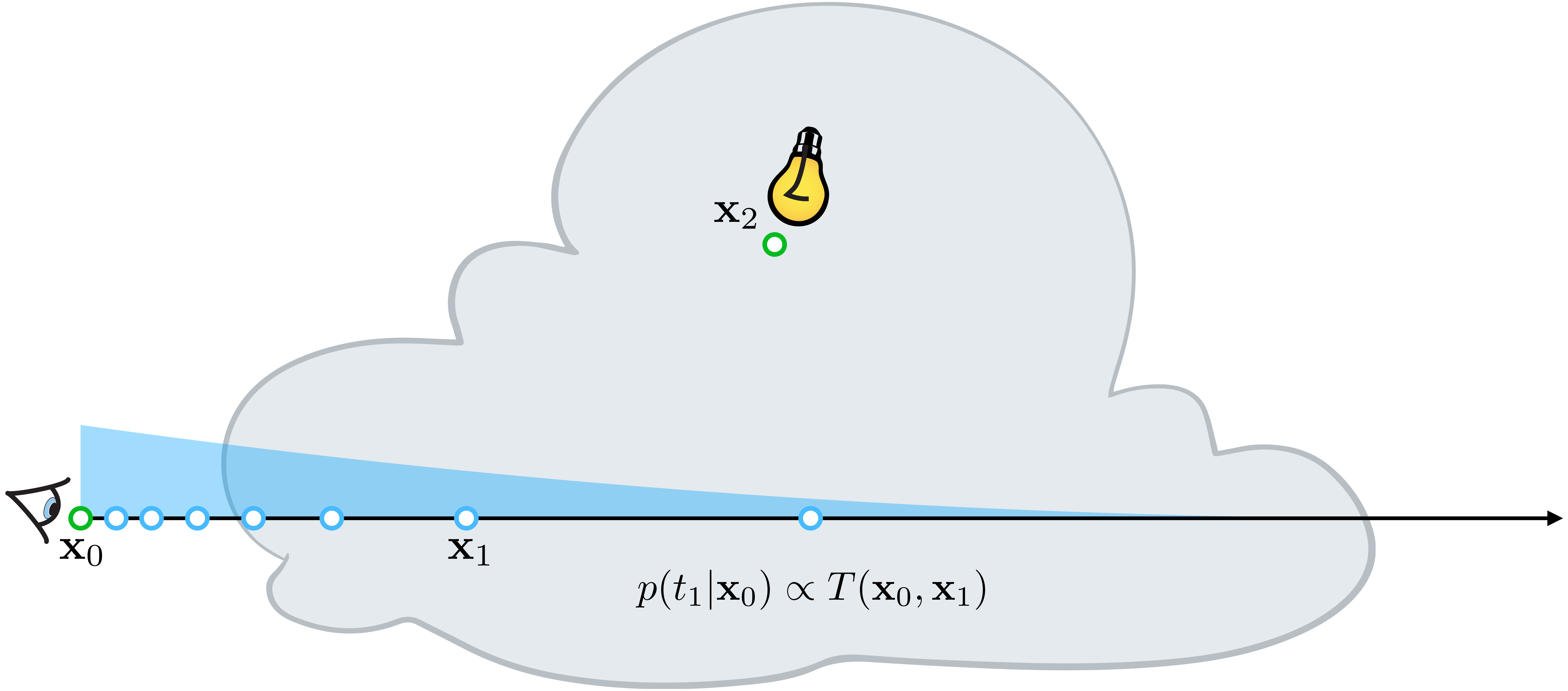
EXPLICIT LIGHT SAMPLING



EXPLICIT LIGHT SAMPLING



EXPLICIT: TRANSMITTANCE



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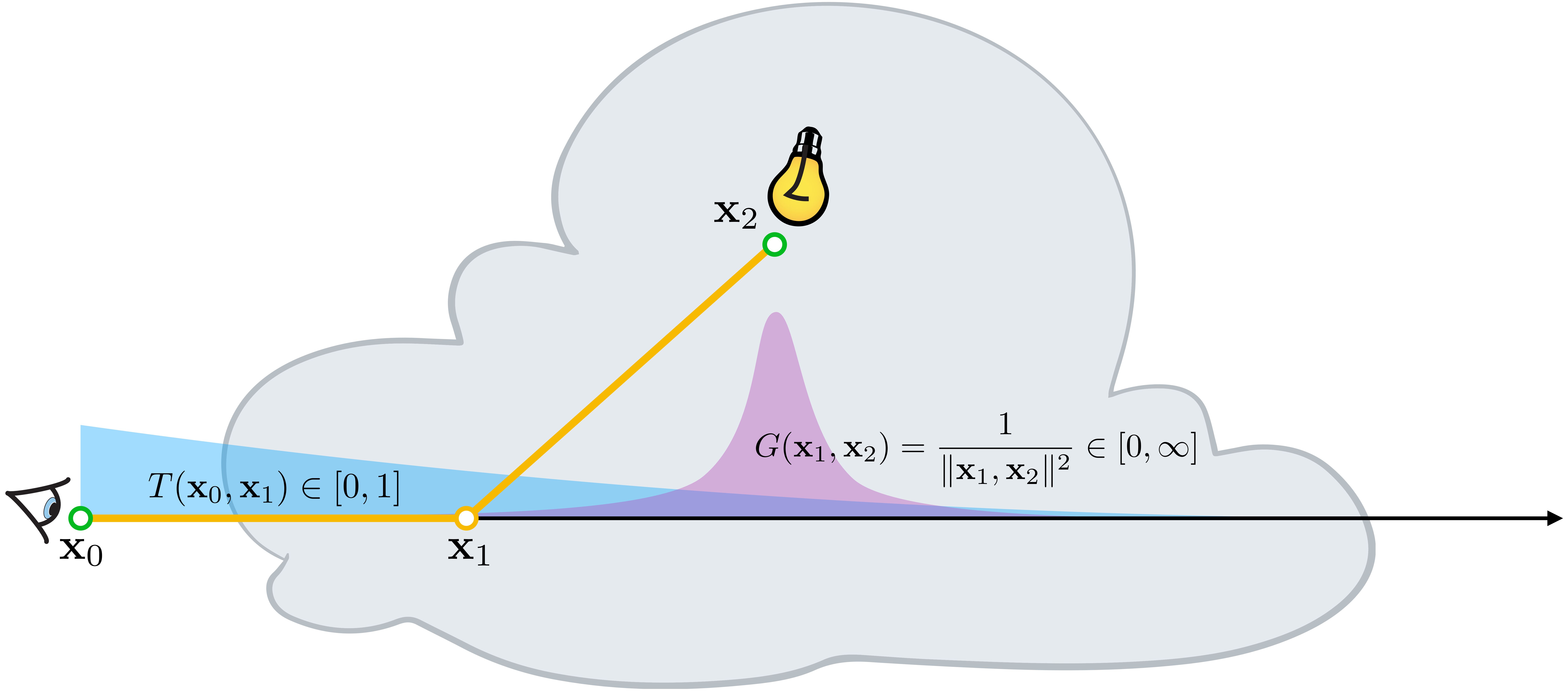
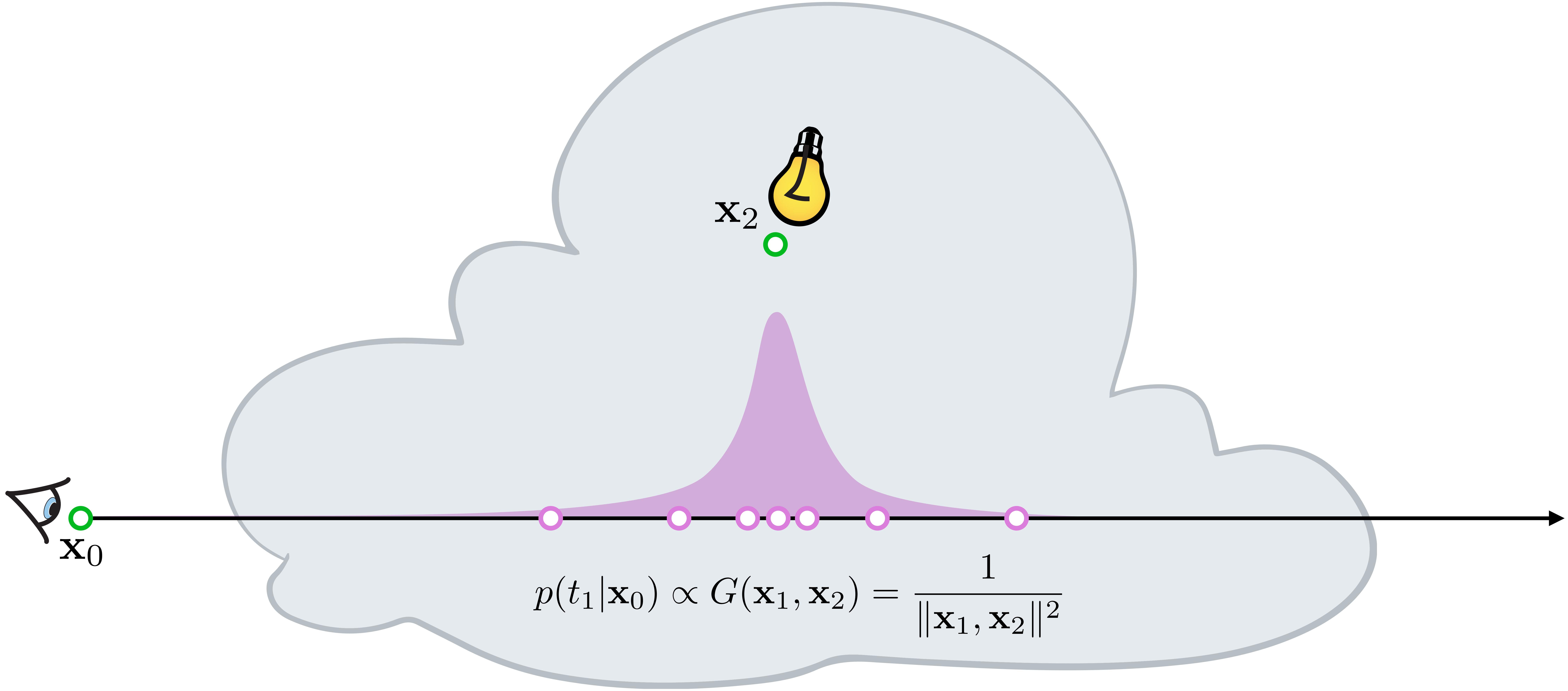
EUROGRAPHICS 2018



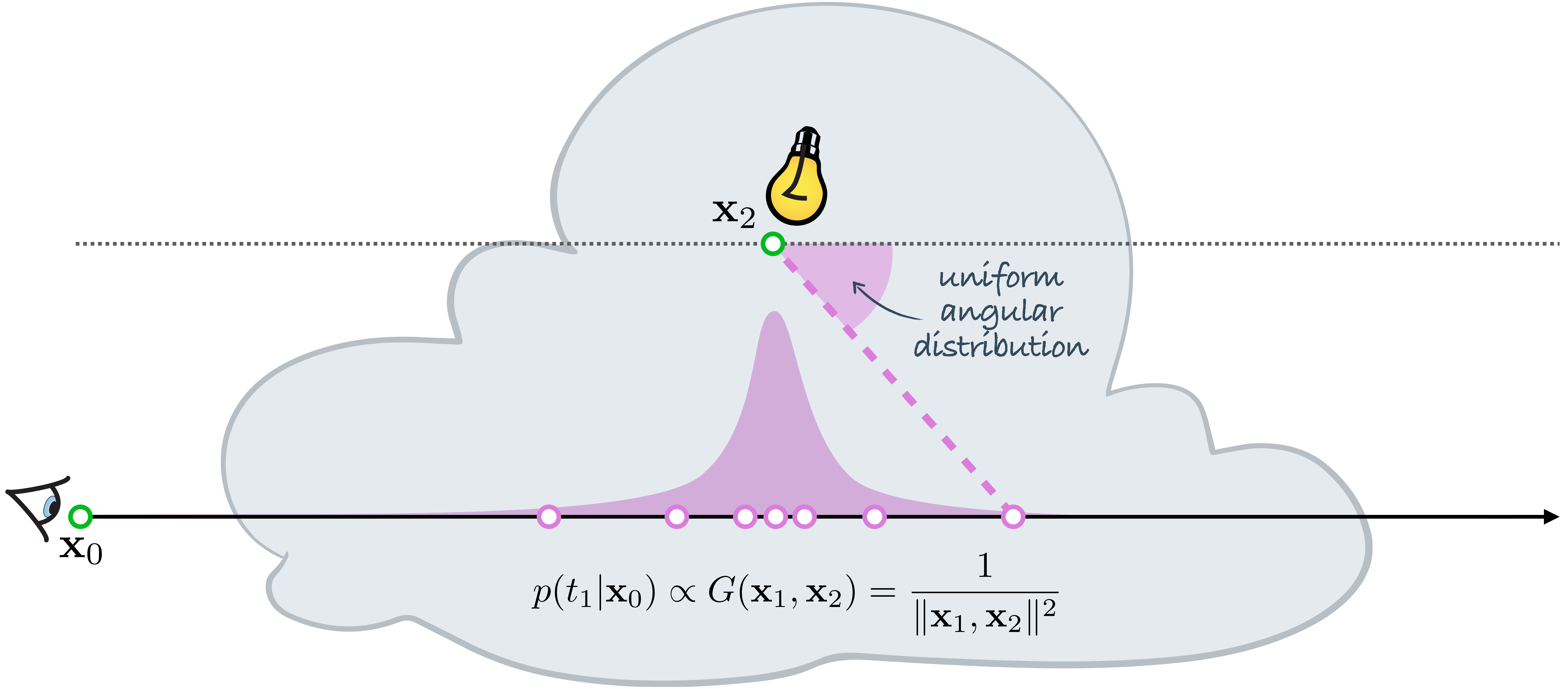
EXPLICIT: EQUIANGULAR



$$p(t_1|\mathbf{x}_0) \propto G(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\|\mathbf{x}_1, \mathbf{x}_2\|^2}$$

EXPLICIT: EQUIANGULAR

EUROGRAPHICS 2018



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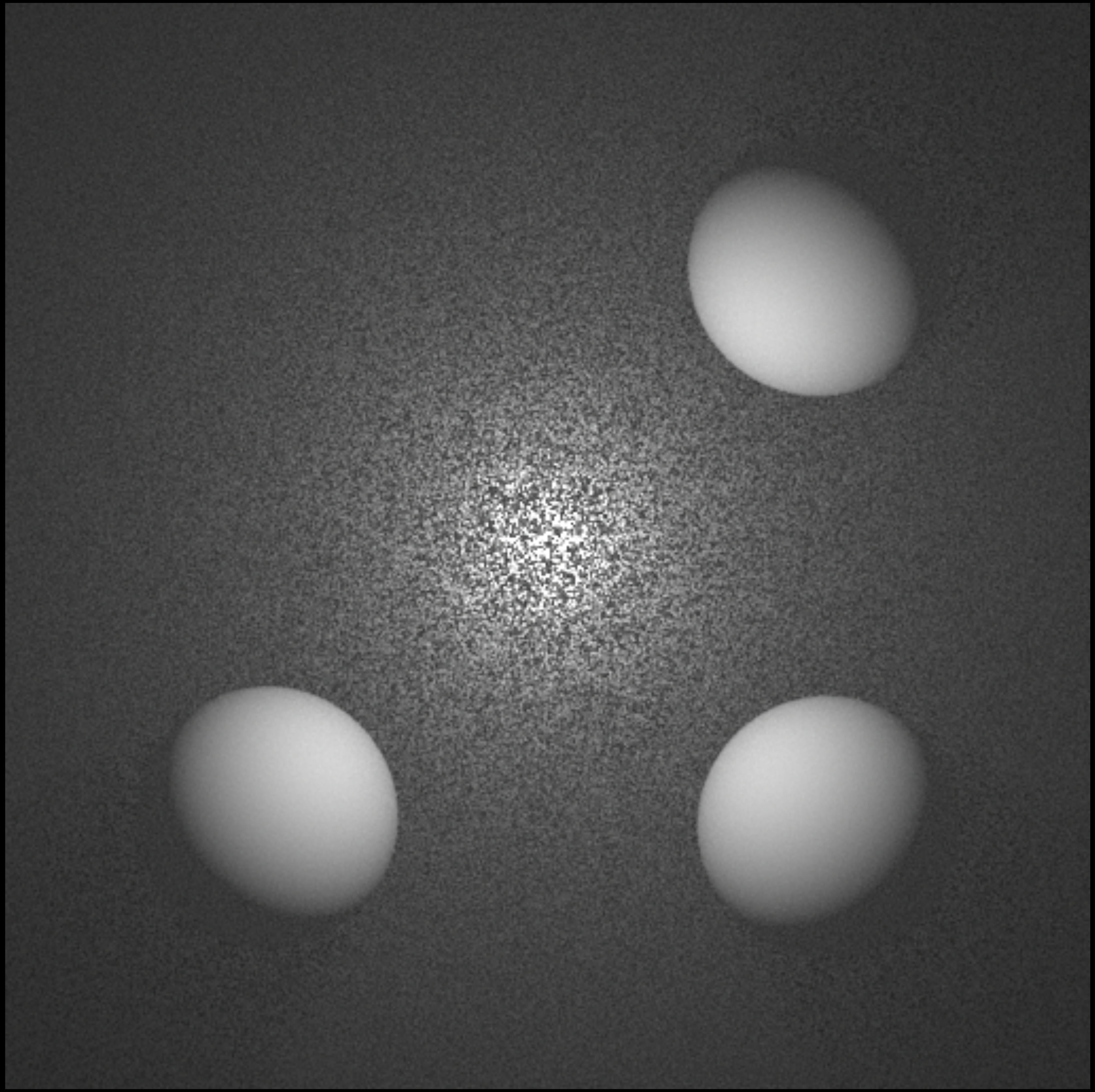
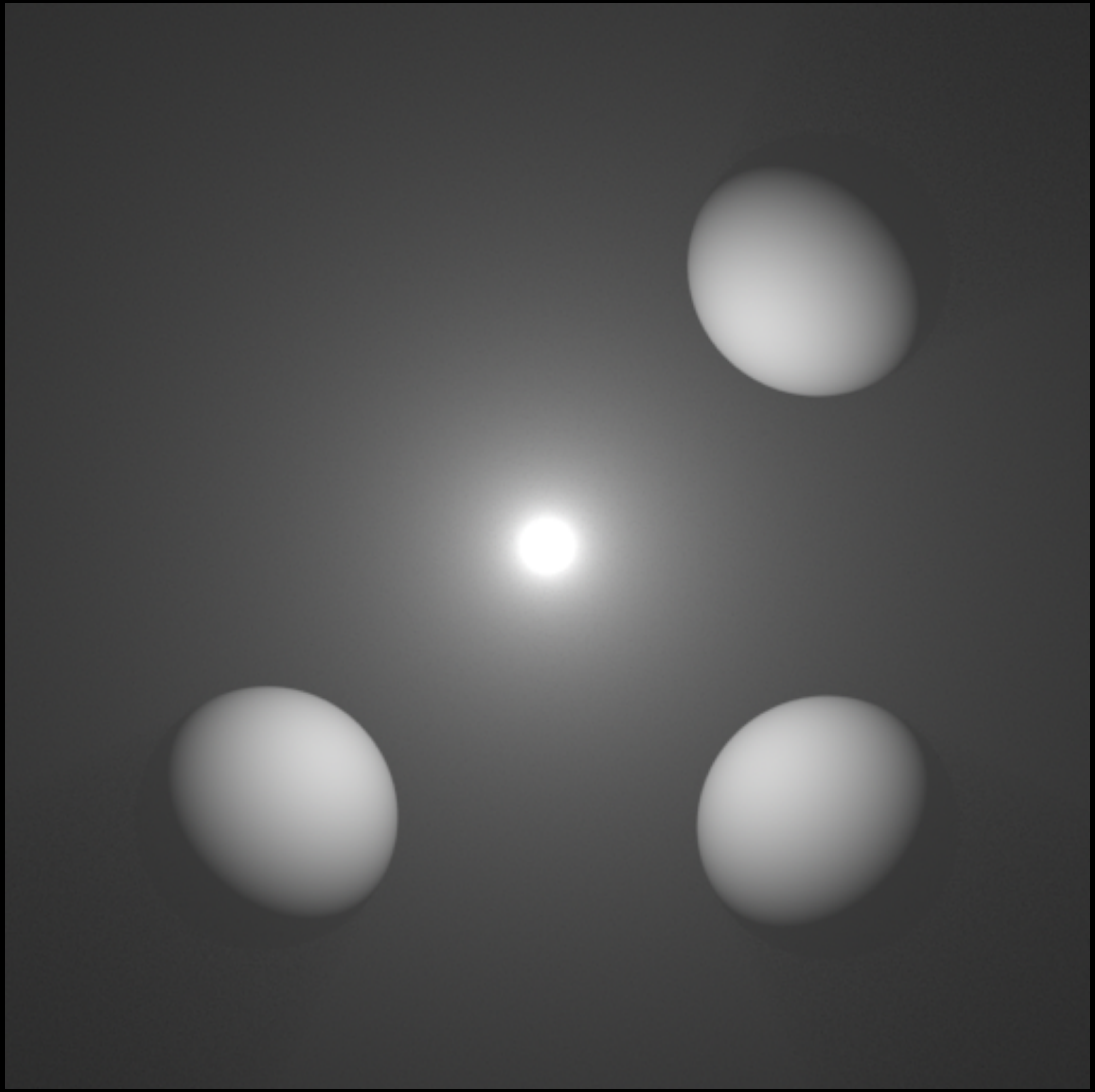


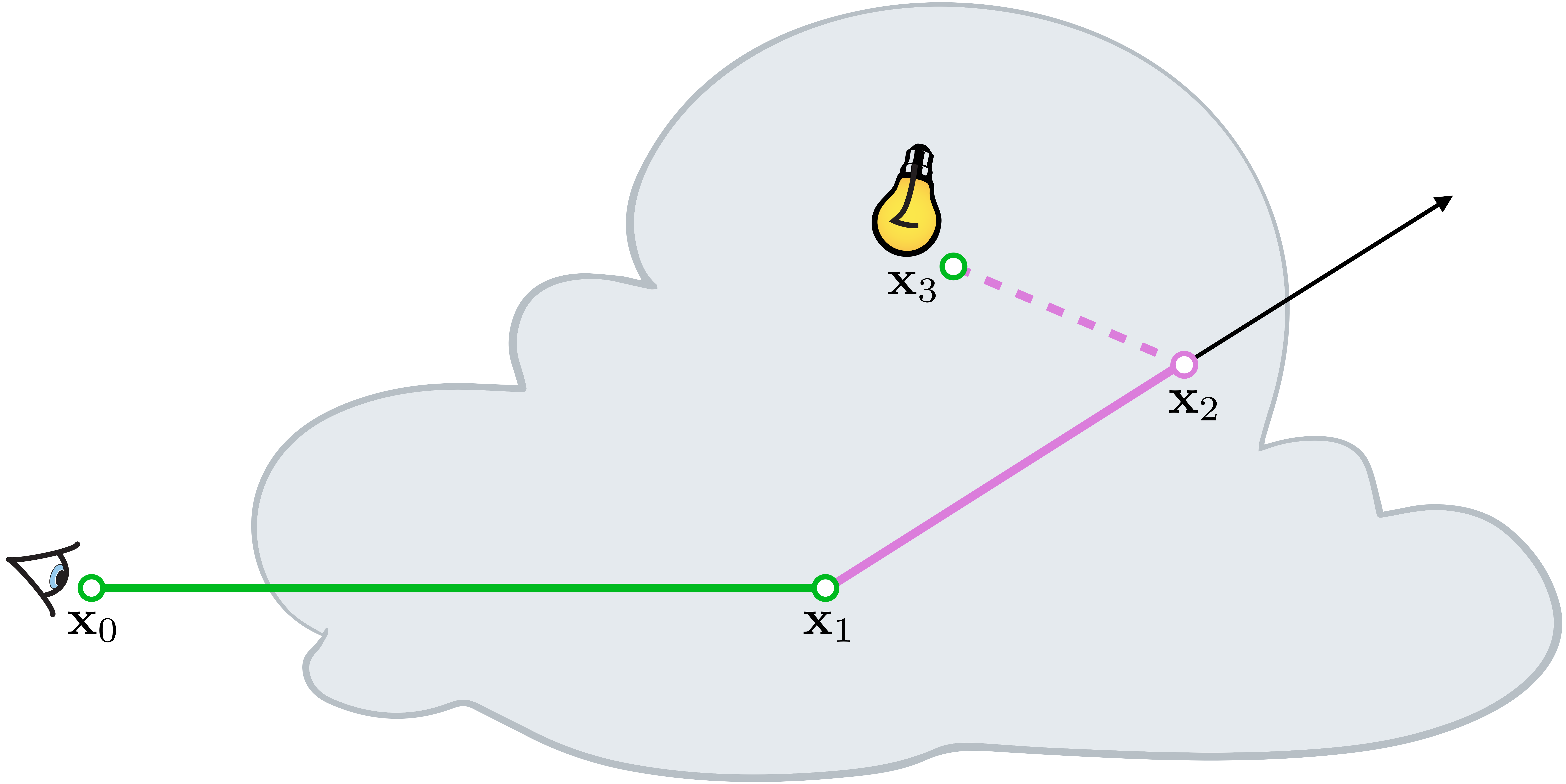
Transmittance sampling, 16 spp

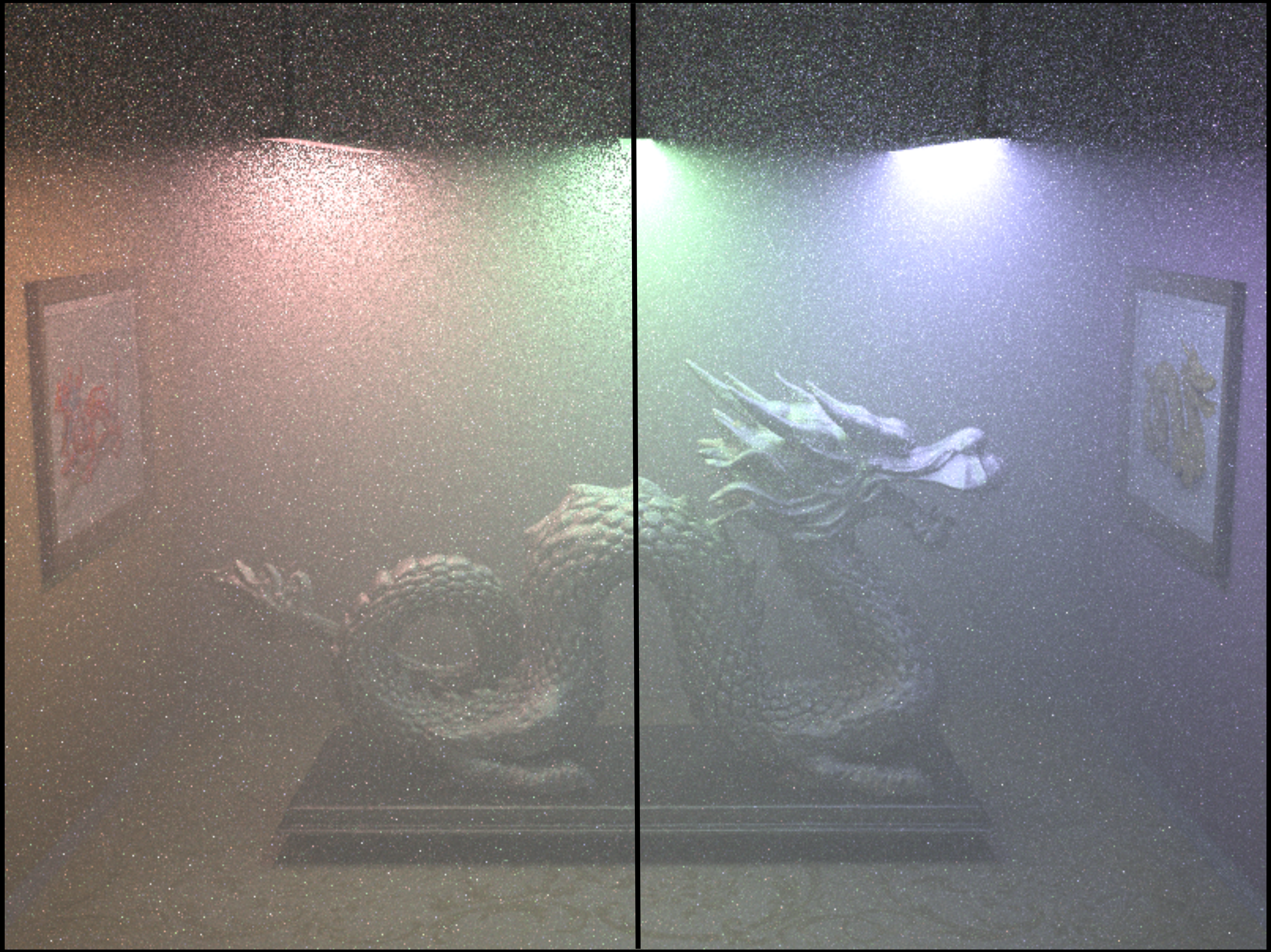


Equiangular sampling, 16 spp

Equiangular sampling Transmittance sampling MIS combination

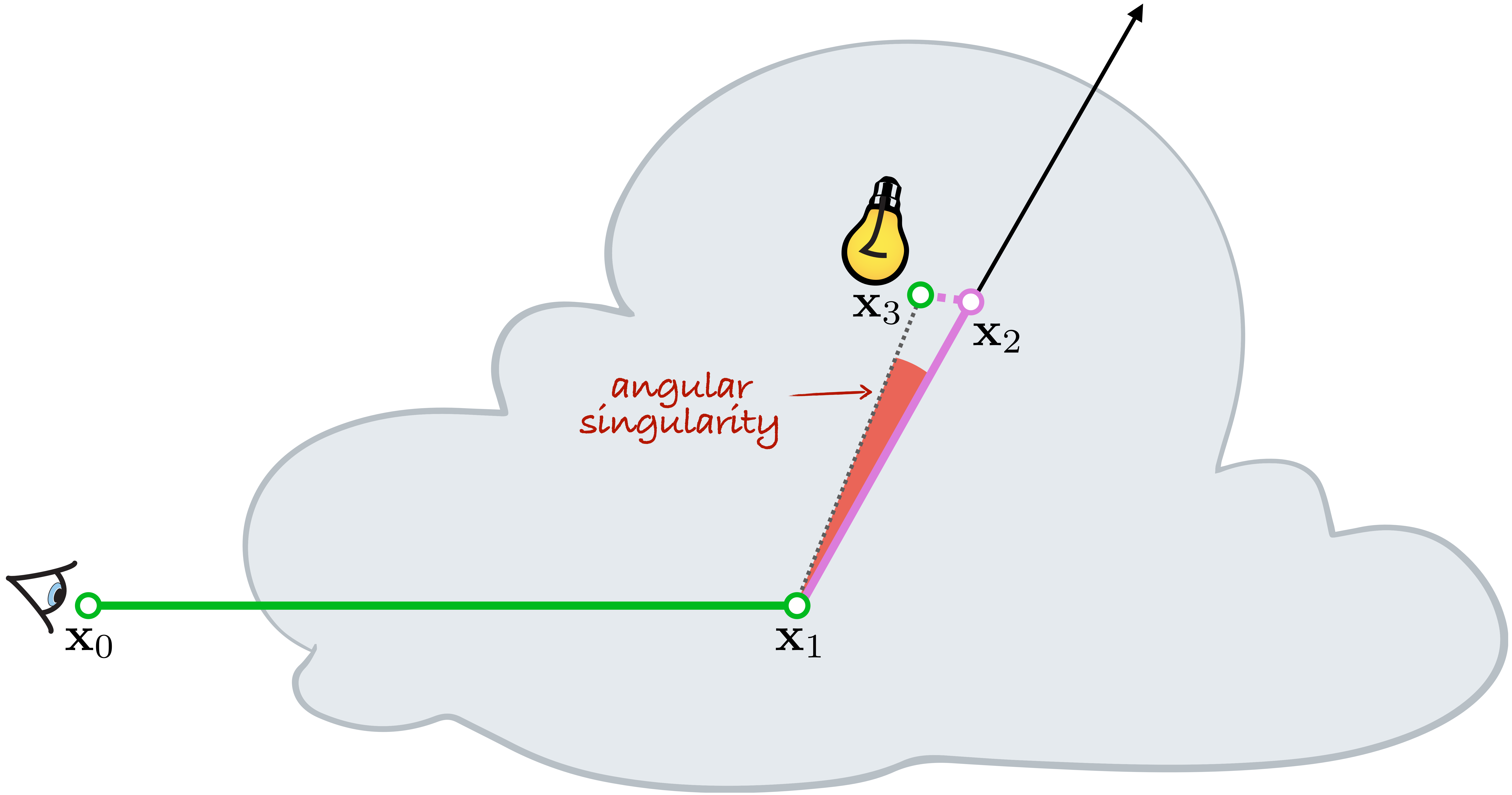
UNIDIRECTIONAL + NEXT EVENT



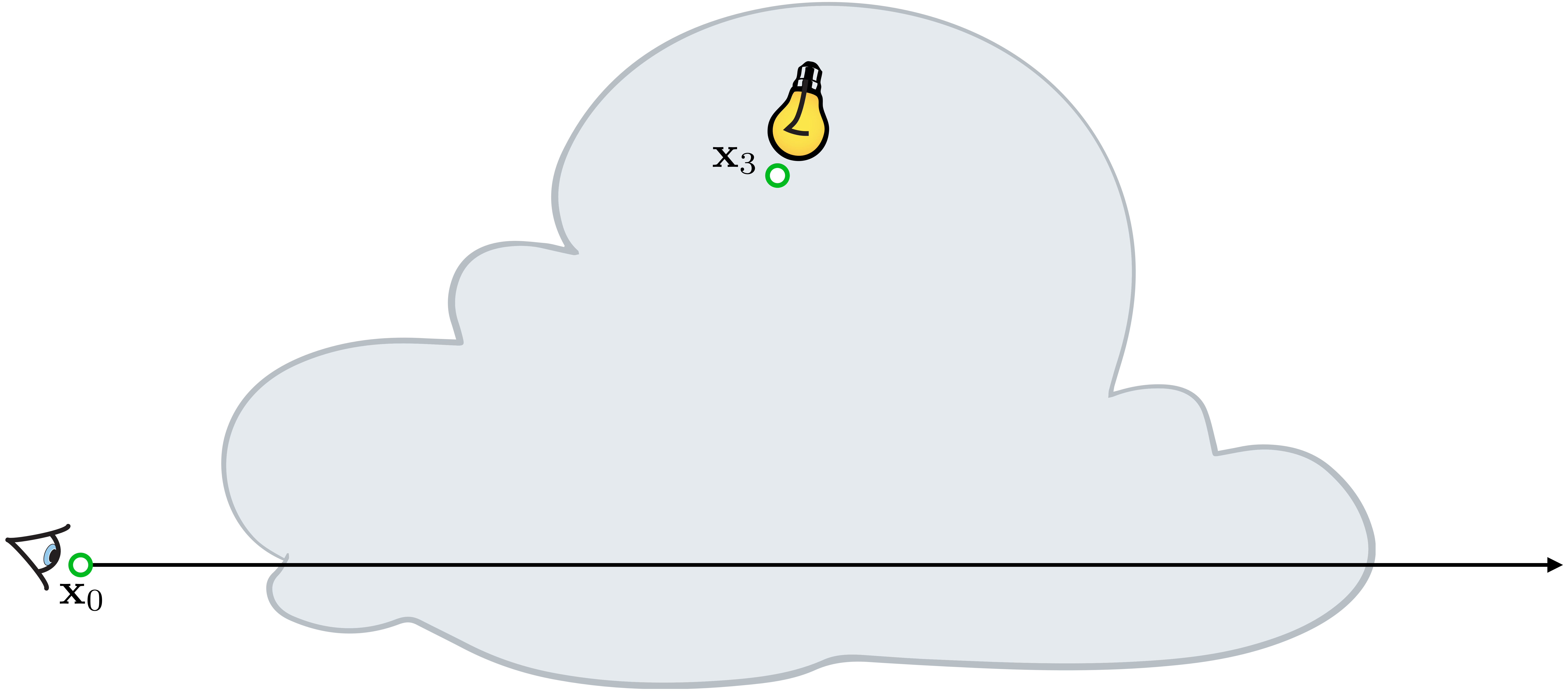


Transmittance connections Equiangular connections

UNIDIRECTIONAL + NEXT EVENT



JOINT PATH SAMPLING



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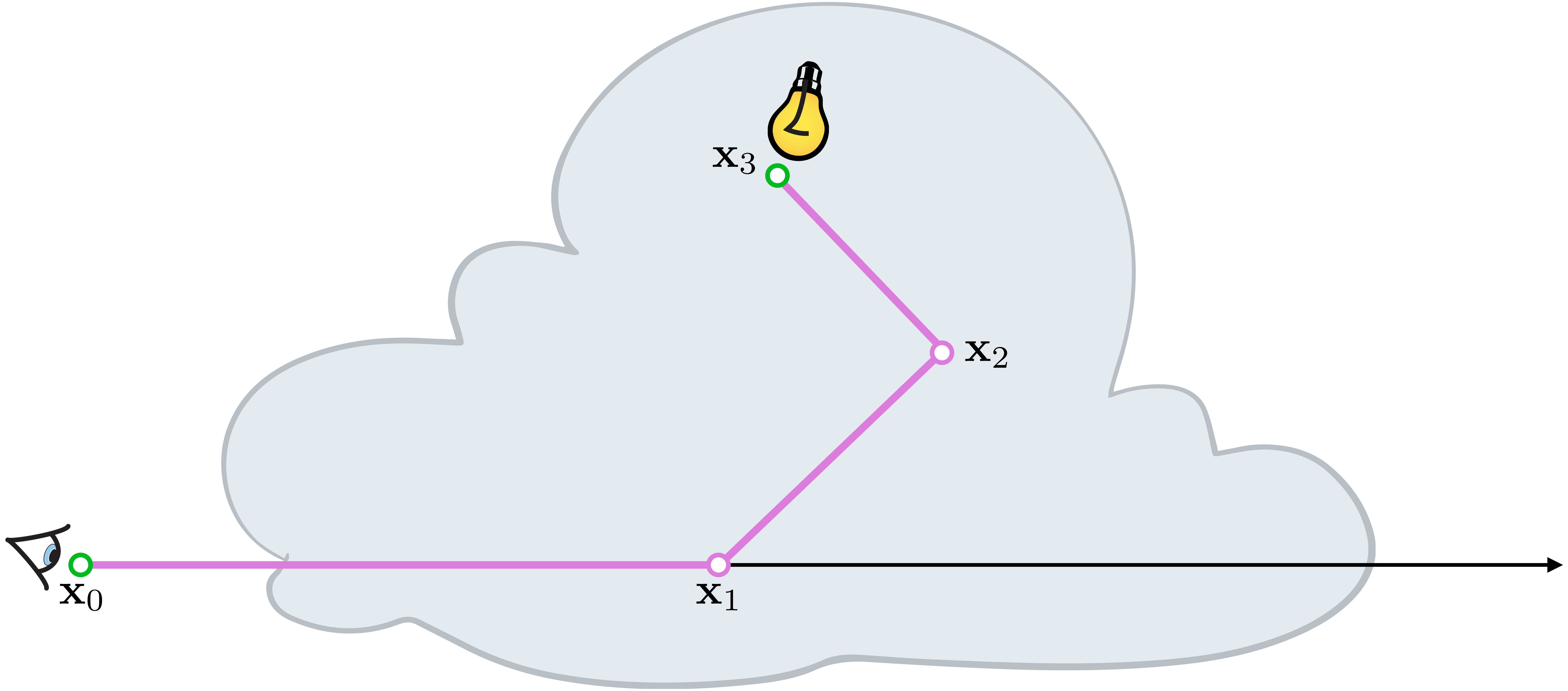

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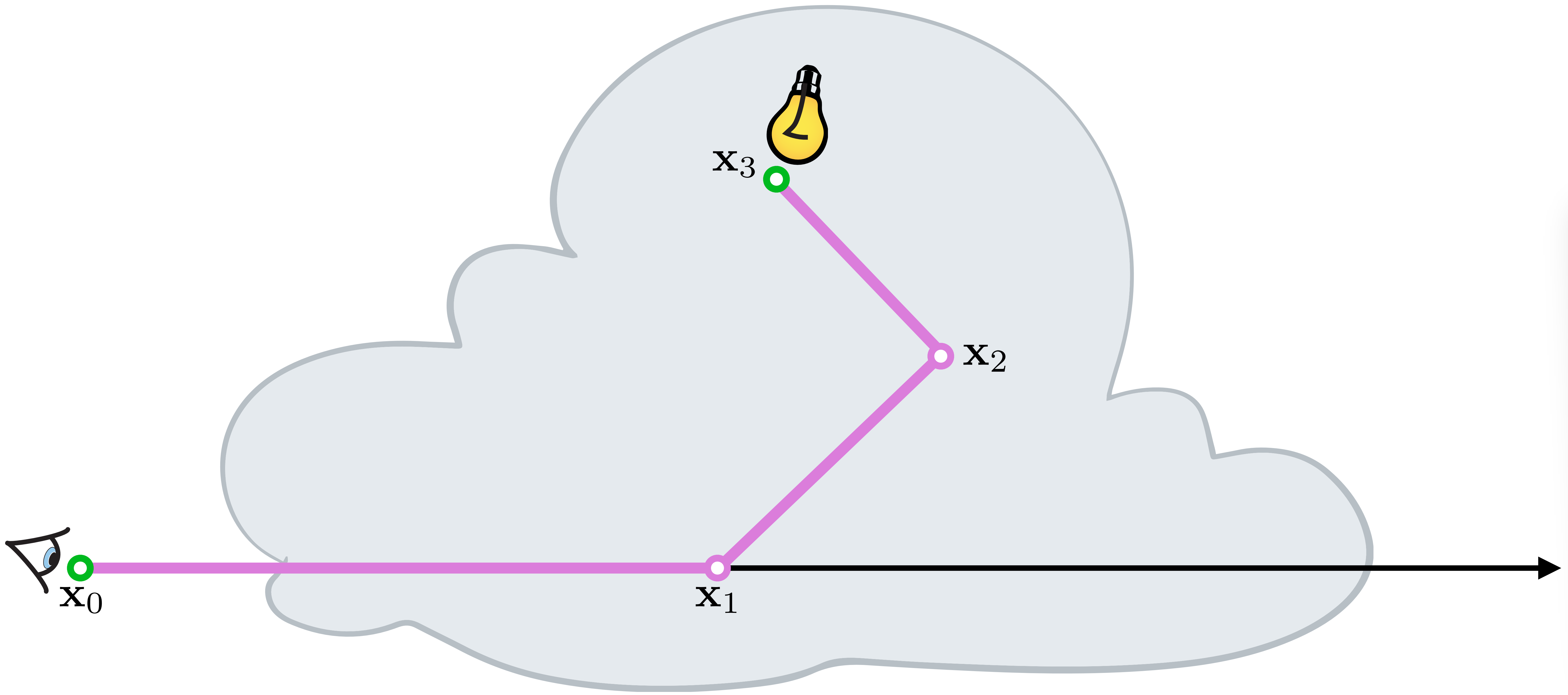


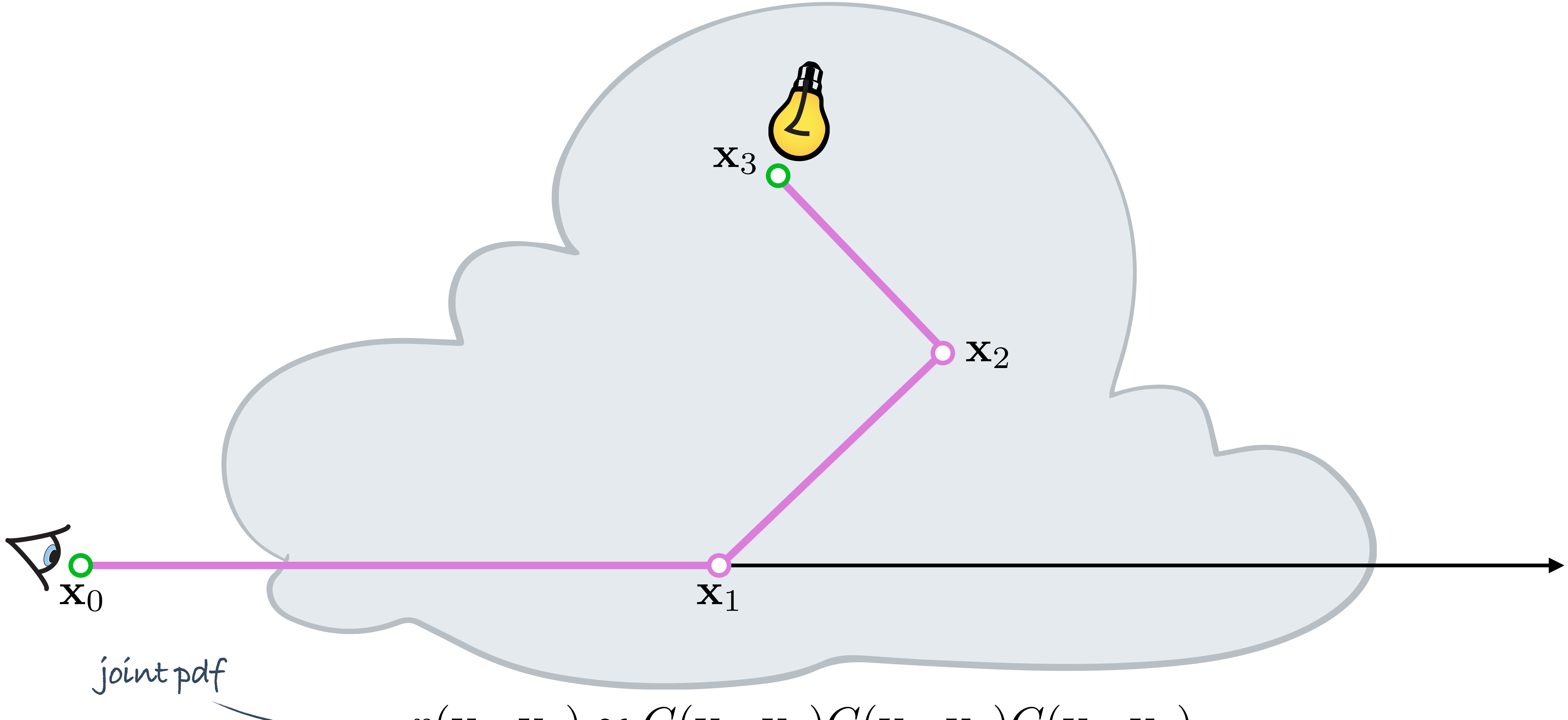
Joint path sampling:

1) Prescribe joint pdf
2) Derive conditional pdfs via successive joint pdf marginalization
3) Conditionals are obtained in reverse order

TRADITIONAL: prescribes conditional pdfs, no explicit control over joint pdf

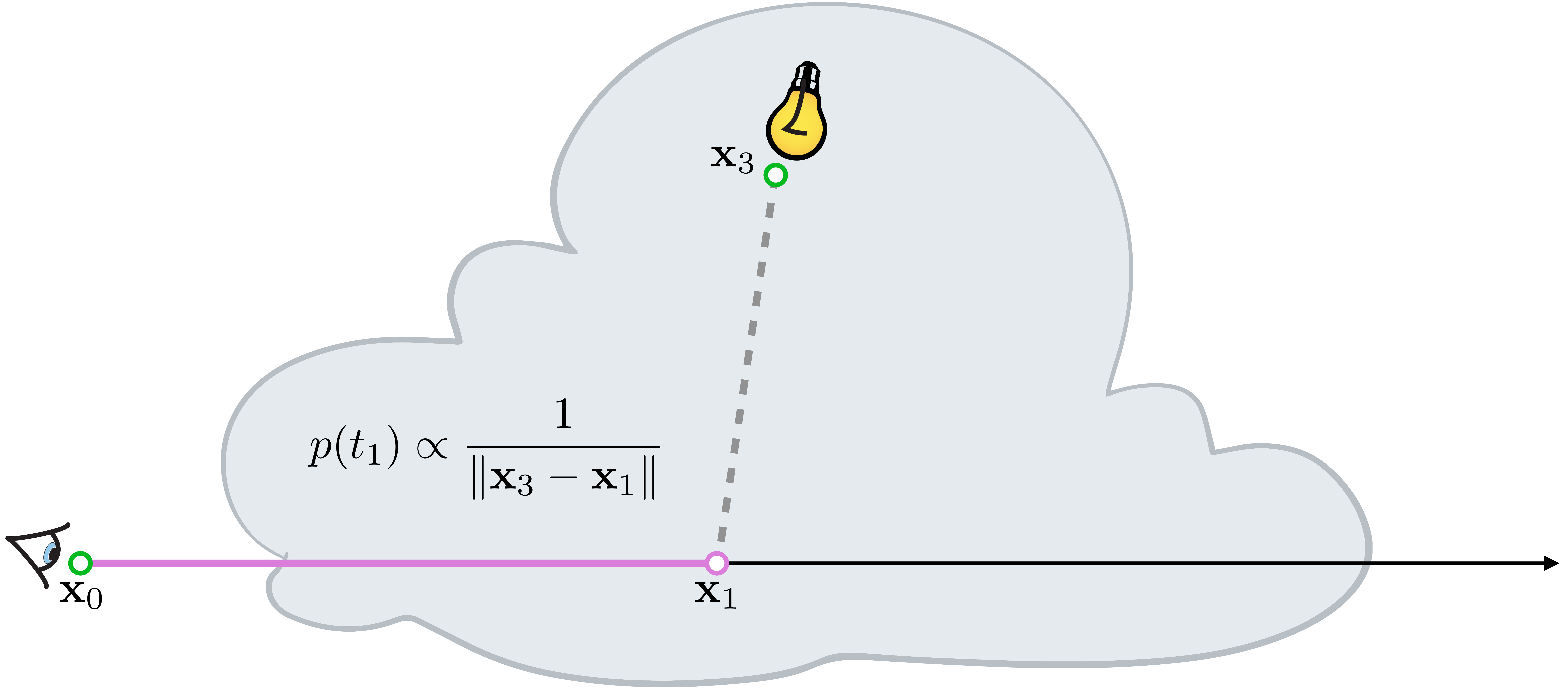
JOINT SAMPLING: prescribe joint pdf, conditional pdfs derived from it

JOINT PATH SAMPLING



$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_2, \mathbf{x}_3)$$

JOINT PATH SAMPLING



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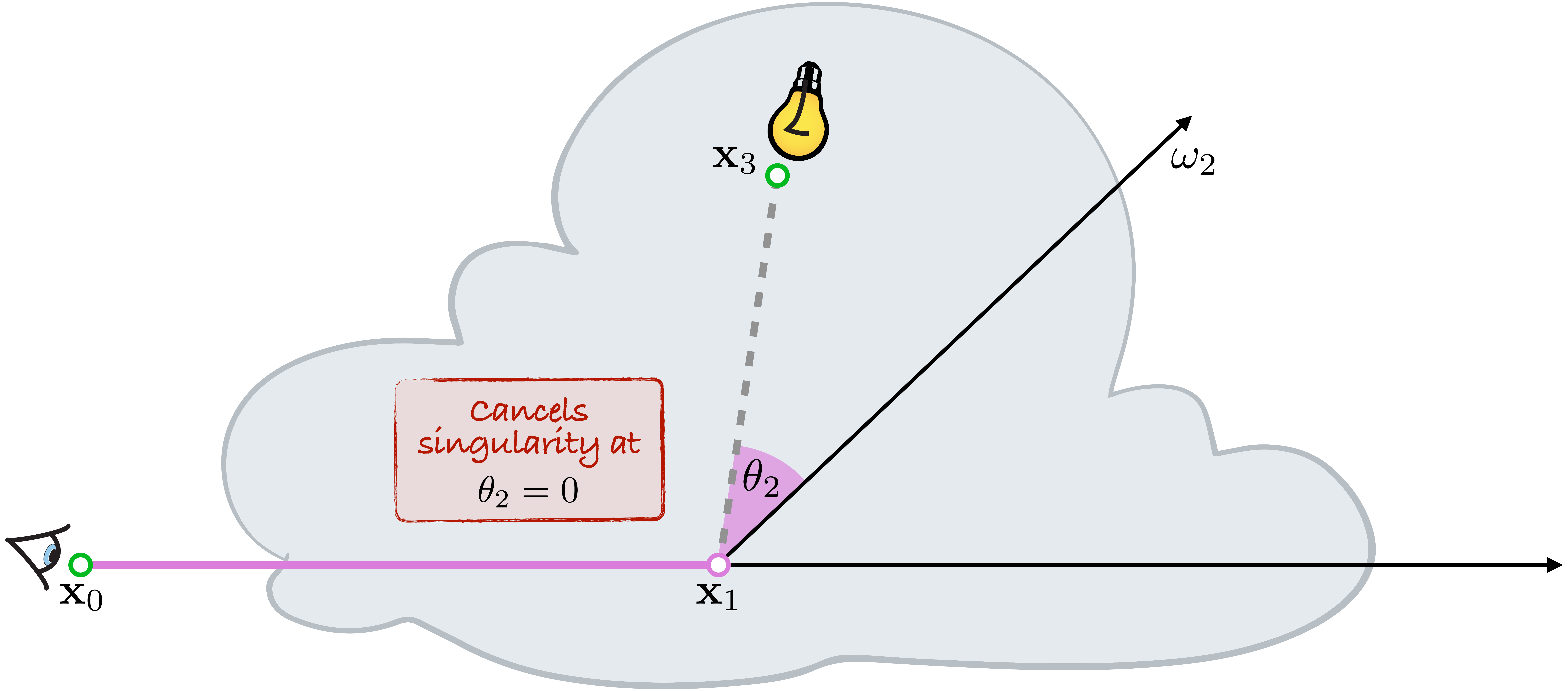

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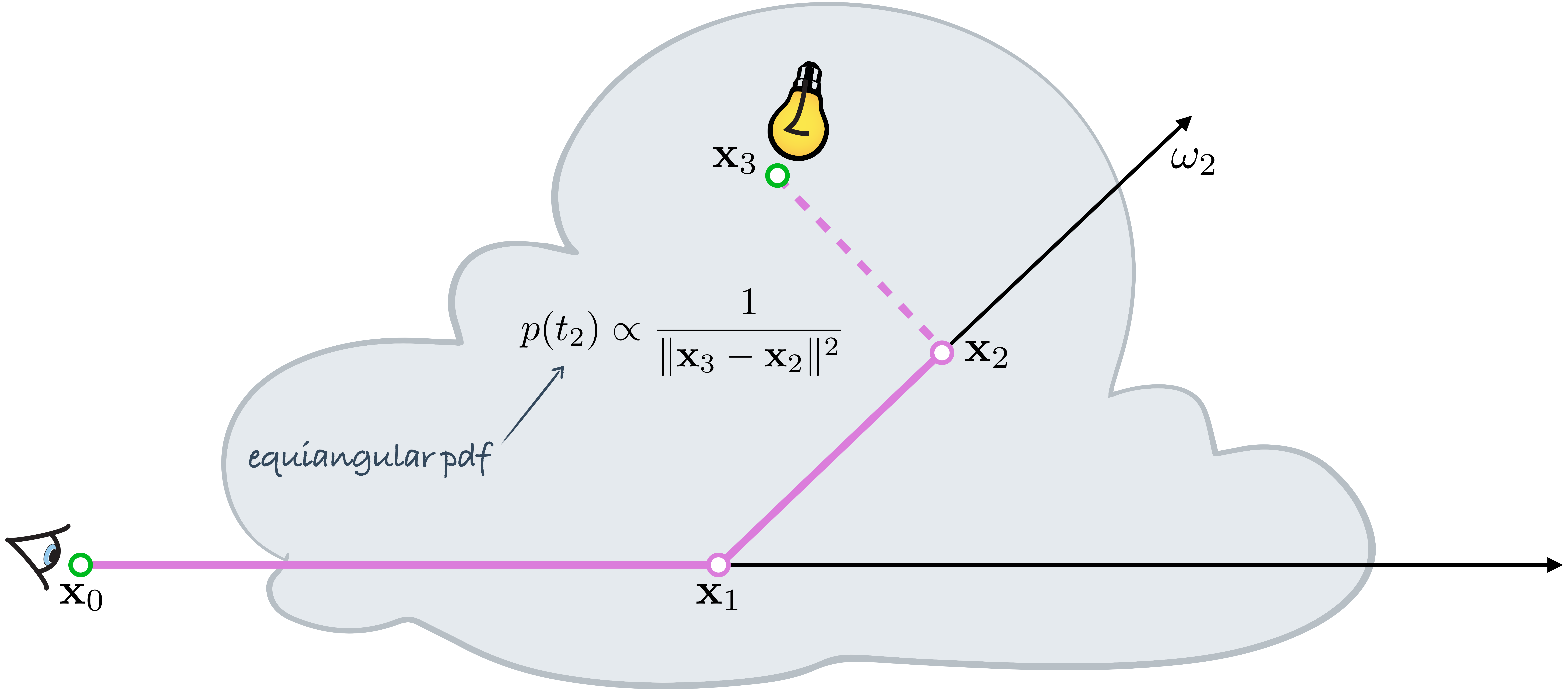

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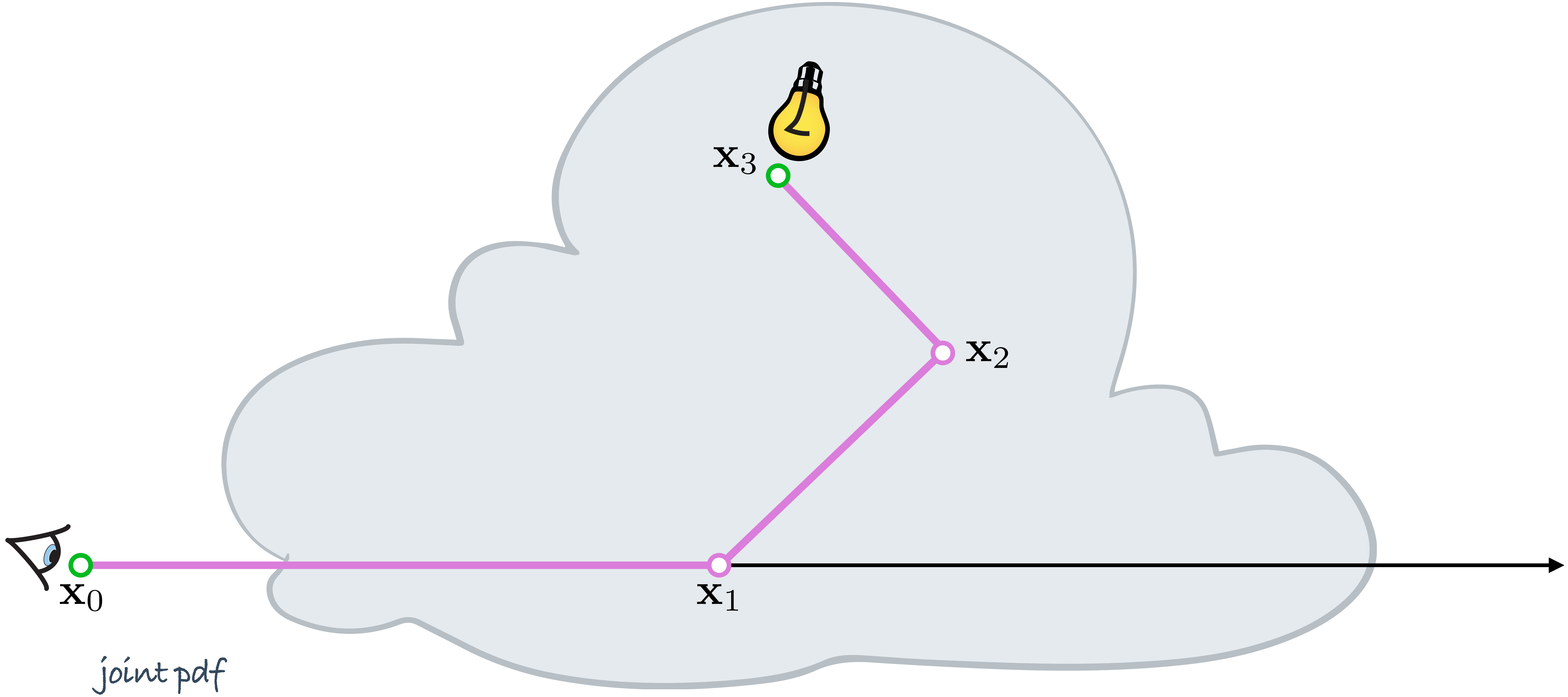

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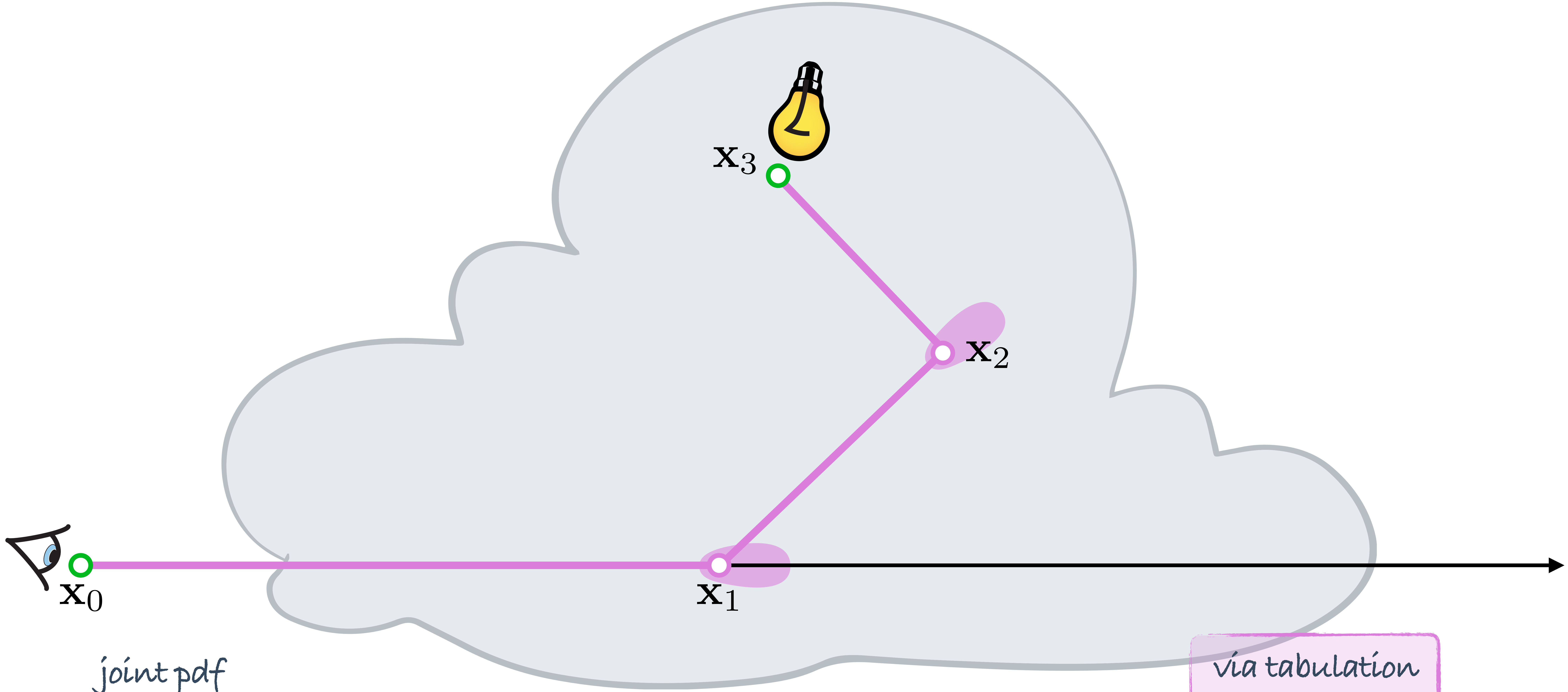
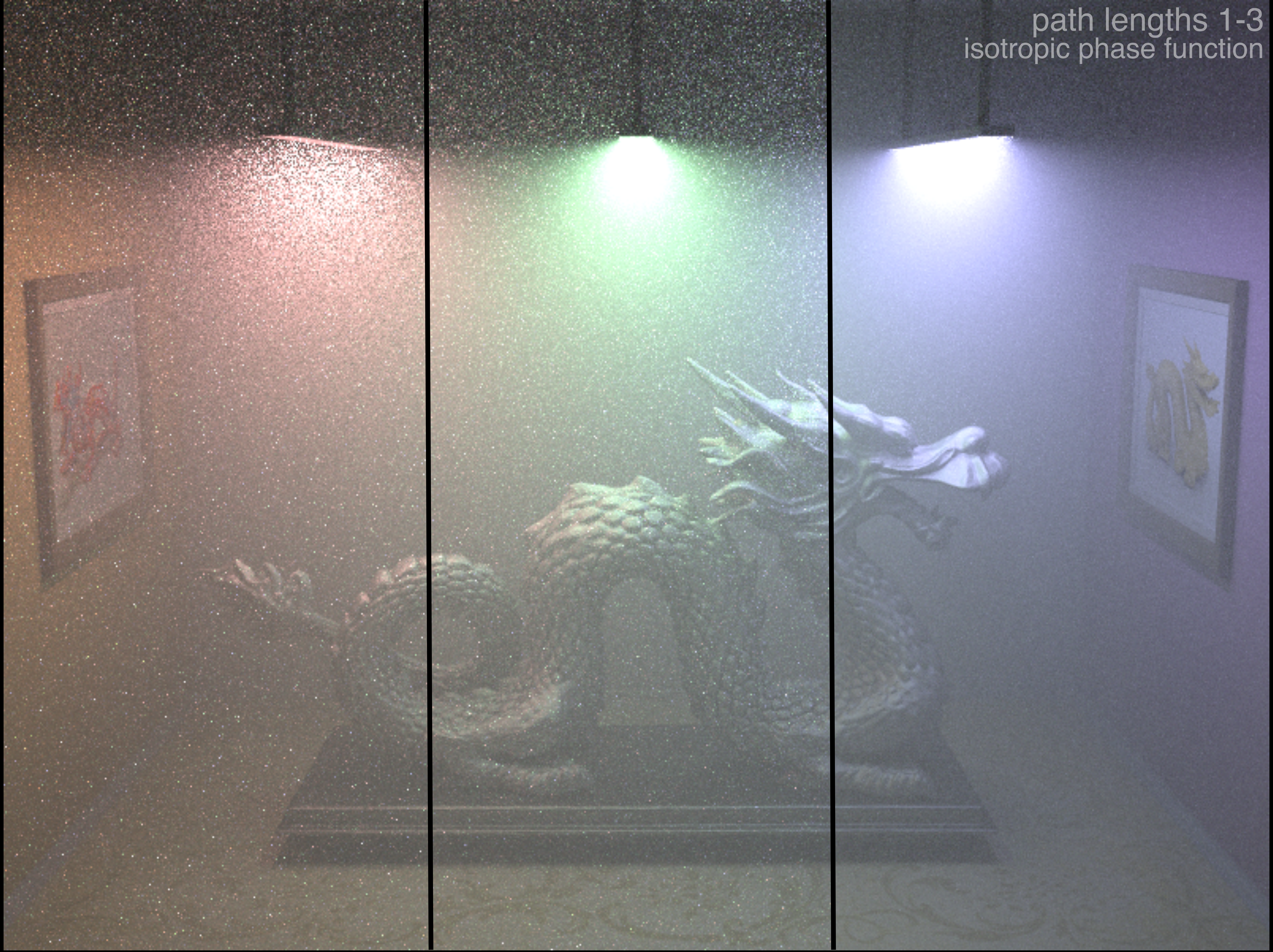
JOINT PATH SAMPLING



joint pdf

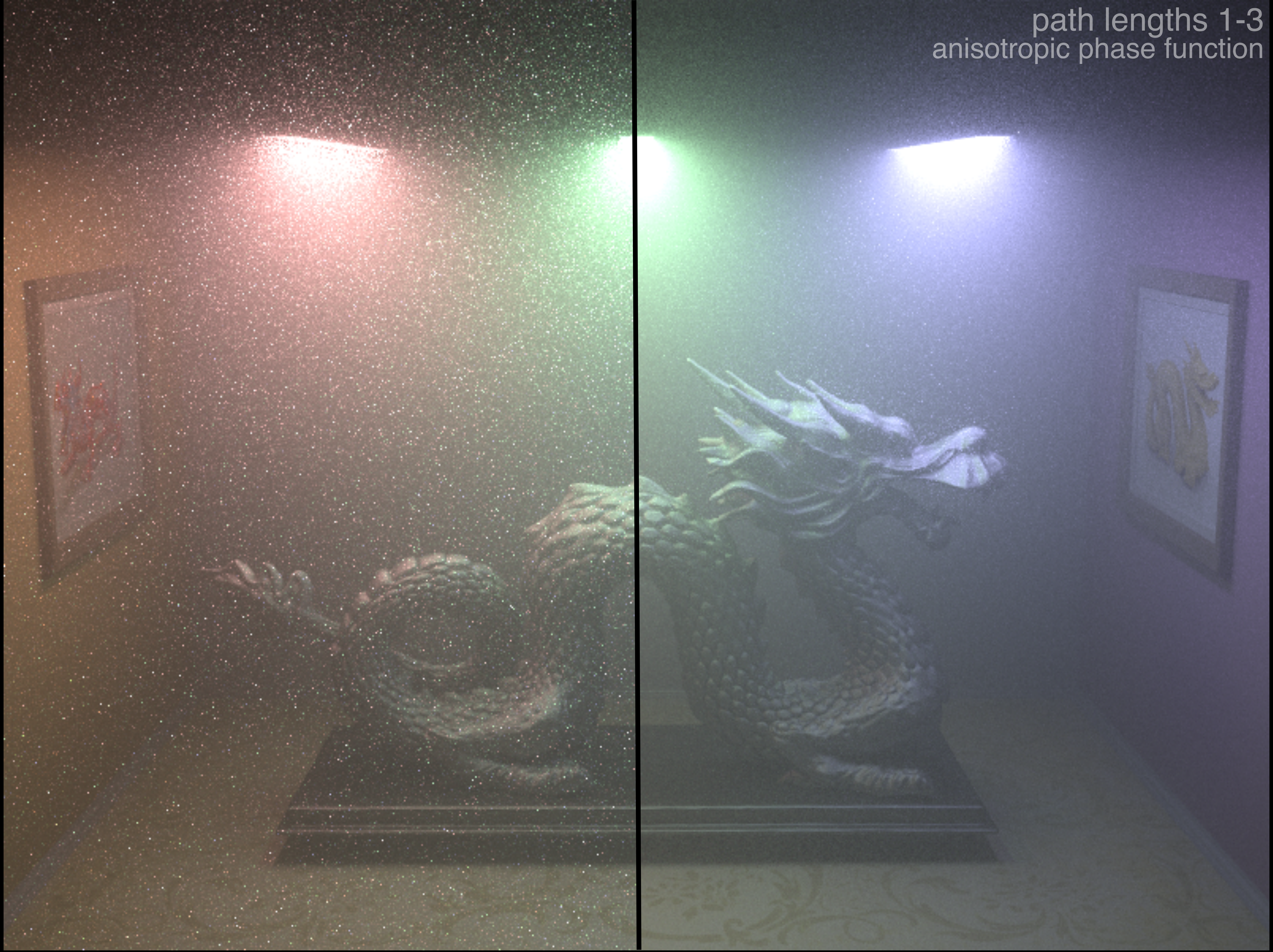
via tabulation

$$p(\mathbf{x}_1, \mathbf{x}_2) \propto G(\mathbf{x}_0, \mathbf{x}_1) G(\mathbf{x}_1, \mathbf{x}_2) G(\mathbf{x}_2, \mathbf{x}_3) f_s(\mathbf{x}_1) f_s(\mathbf{x}_2)$$



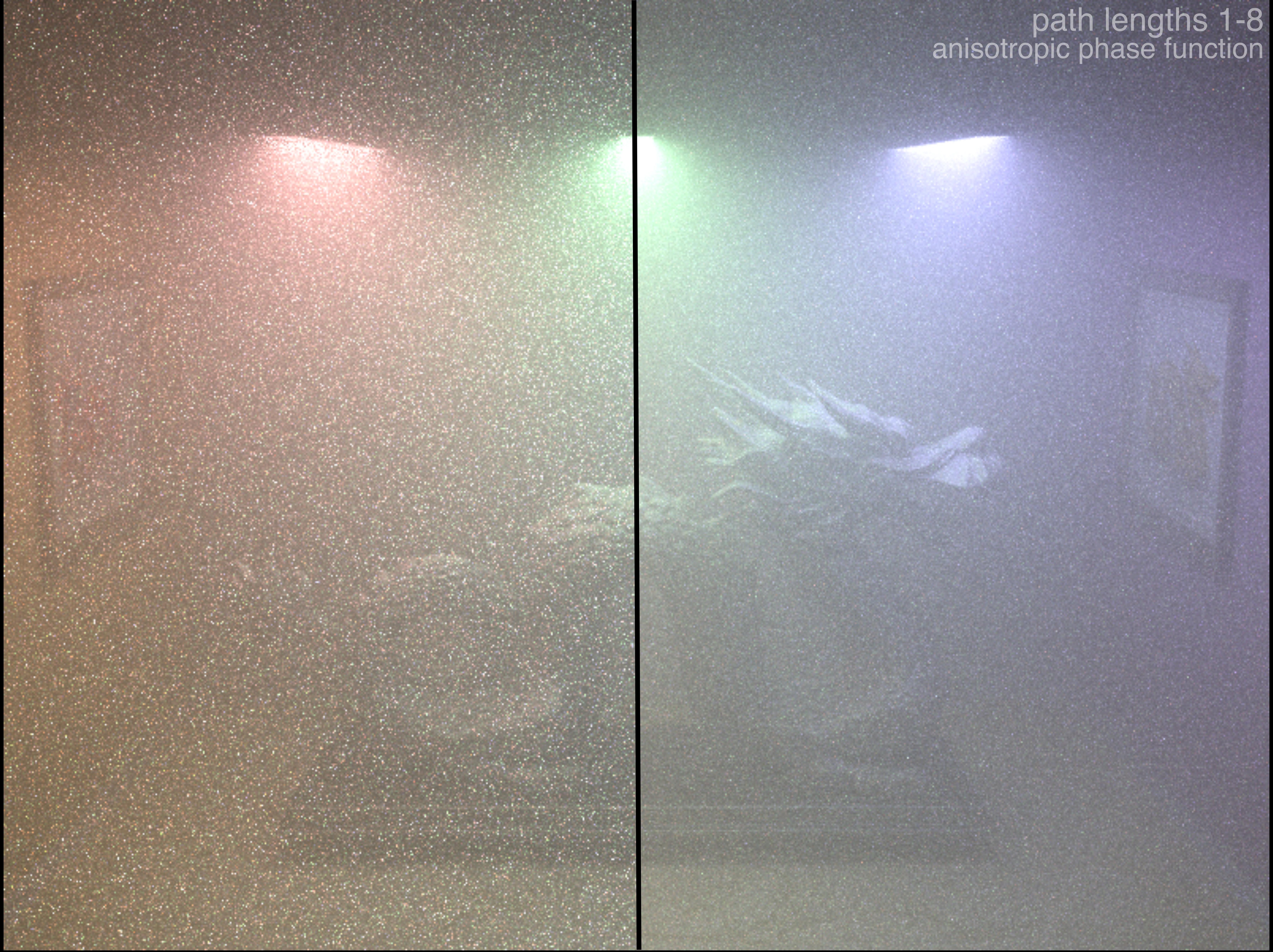
Transmittance Equiangular Joint sampling

Transmittance Equiangular Joint sampling



Transmittance connections

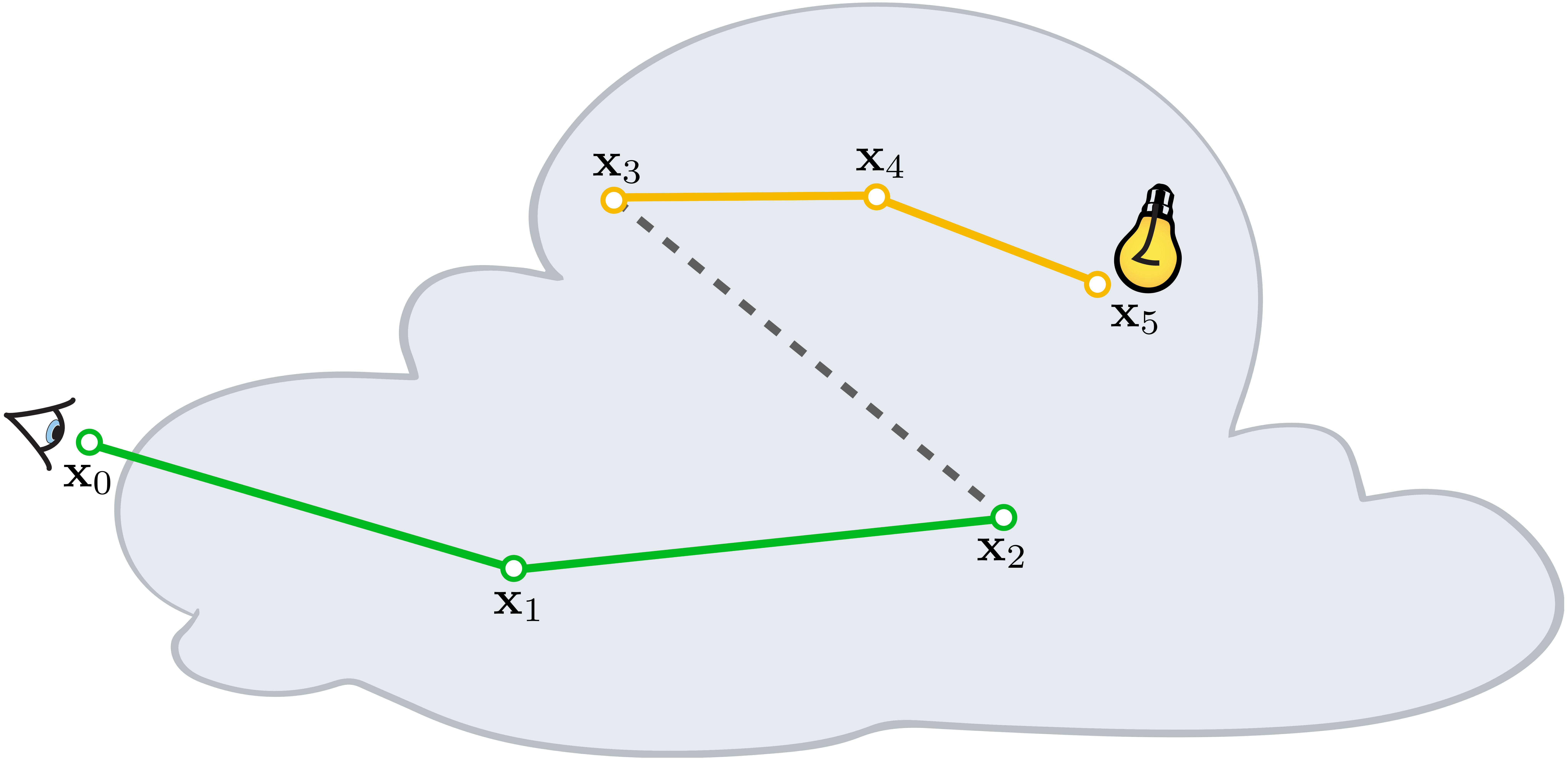
Joint tabulated path sampling



Transmittance connections

Joint tabulated path sampling

BIDIRECTIONAL PATH TRACING



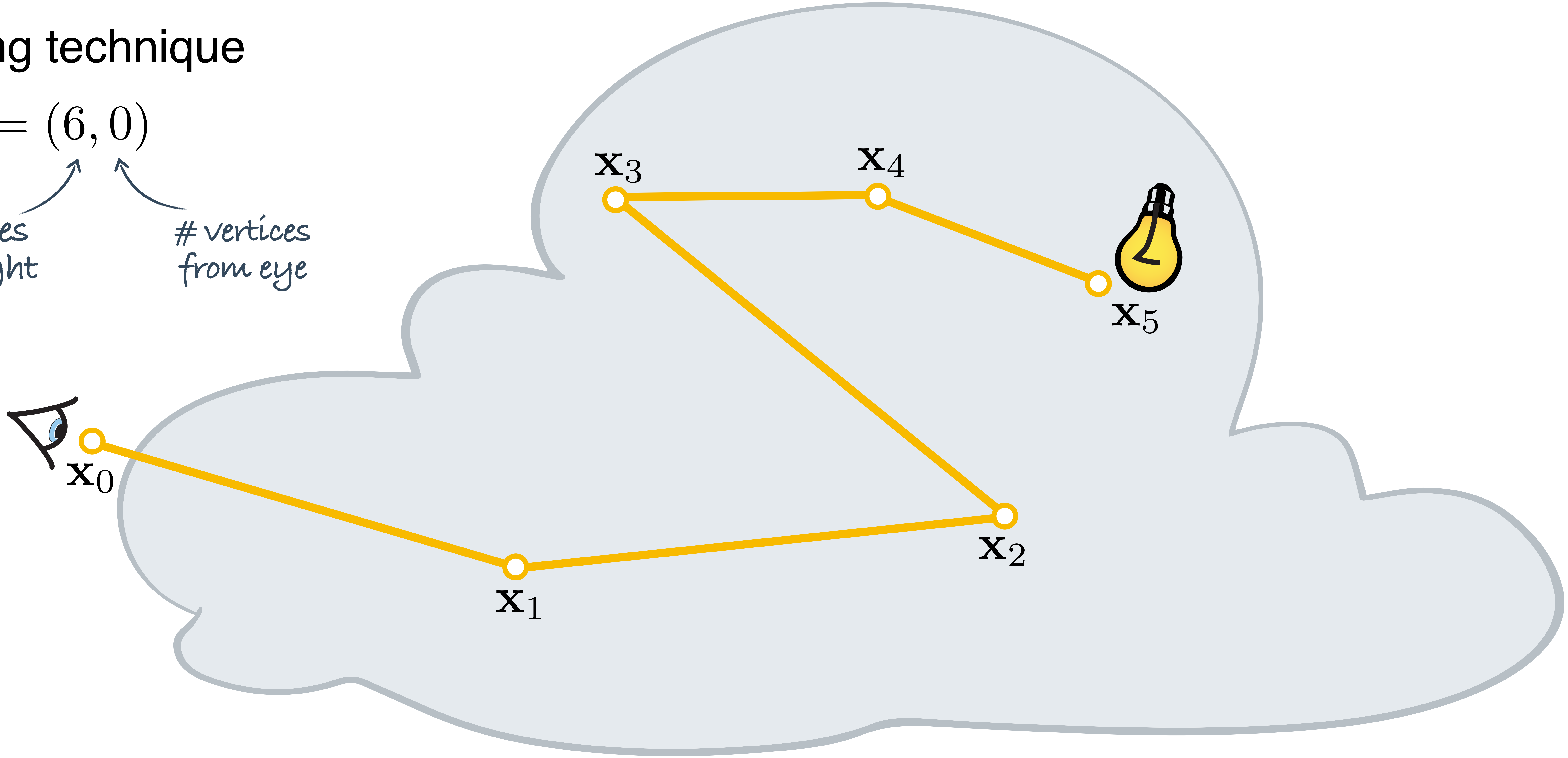
BIDIRECTIONAL PATH TRACING

Sampling technique

$$(s, t) = (6, 0)$$

\# vertices from light

\# vertices from eye

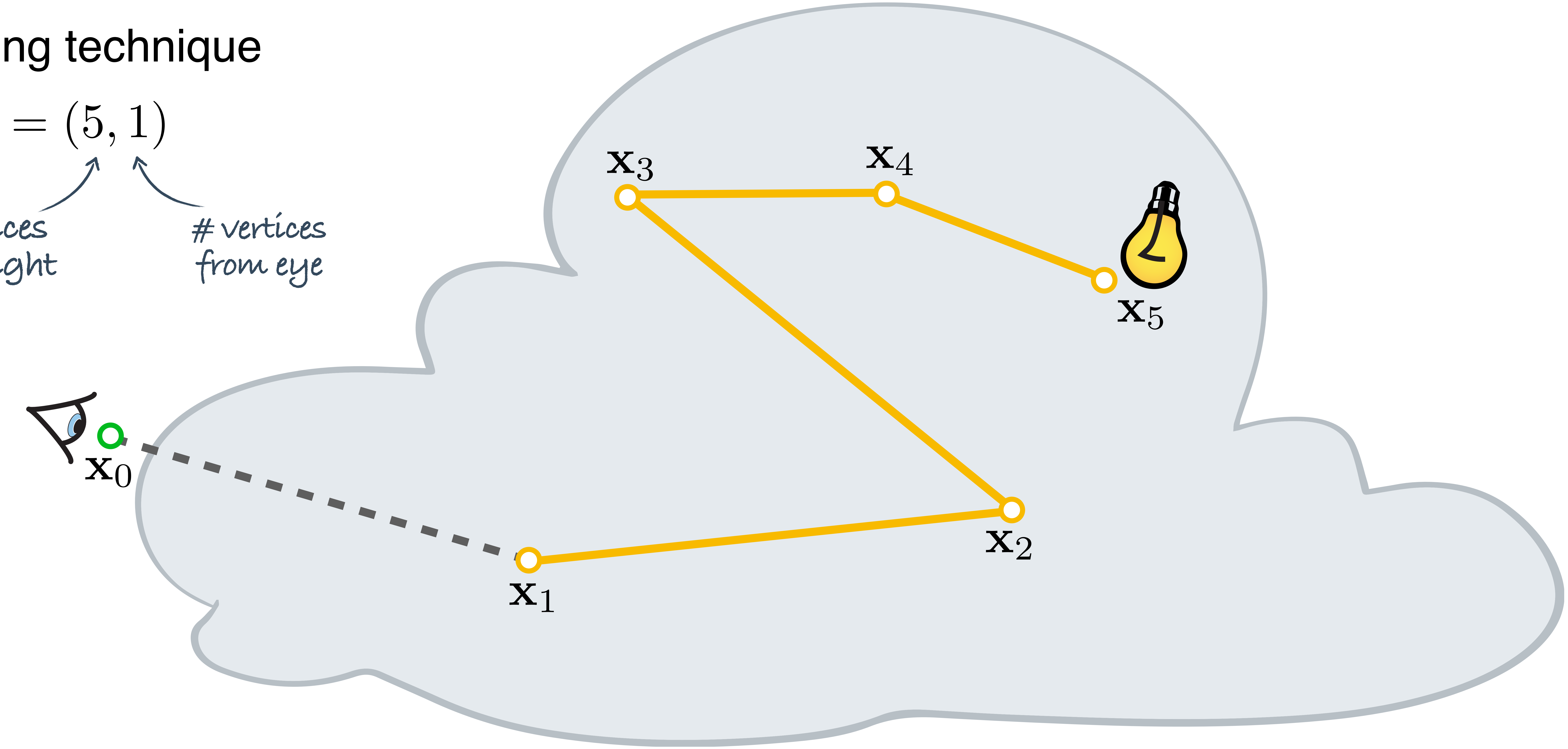


Sampling technique

$$(s,t) = (5,1)$$

\# vertices from light

\# vertices from eye



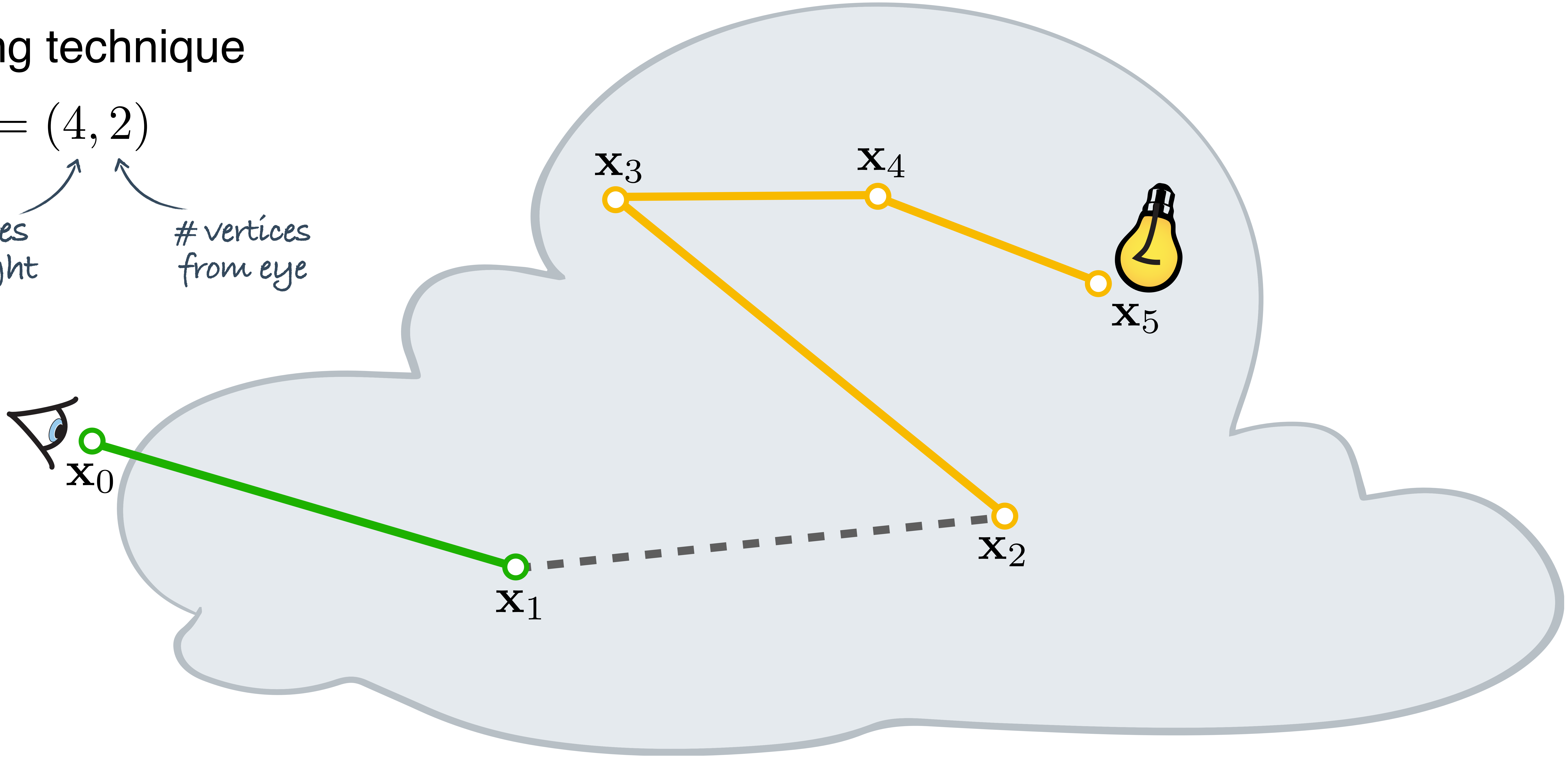
BIDIRECTIONAL PATH TRACING

Sampling technique

$$(s, t) = (4, 2)$$

vertices from light

vertices from eye



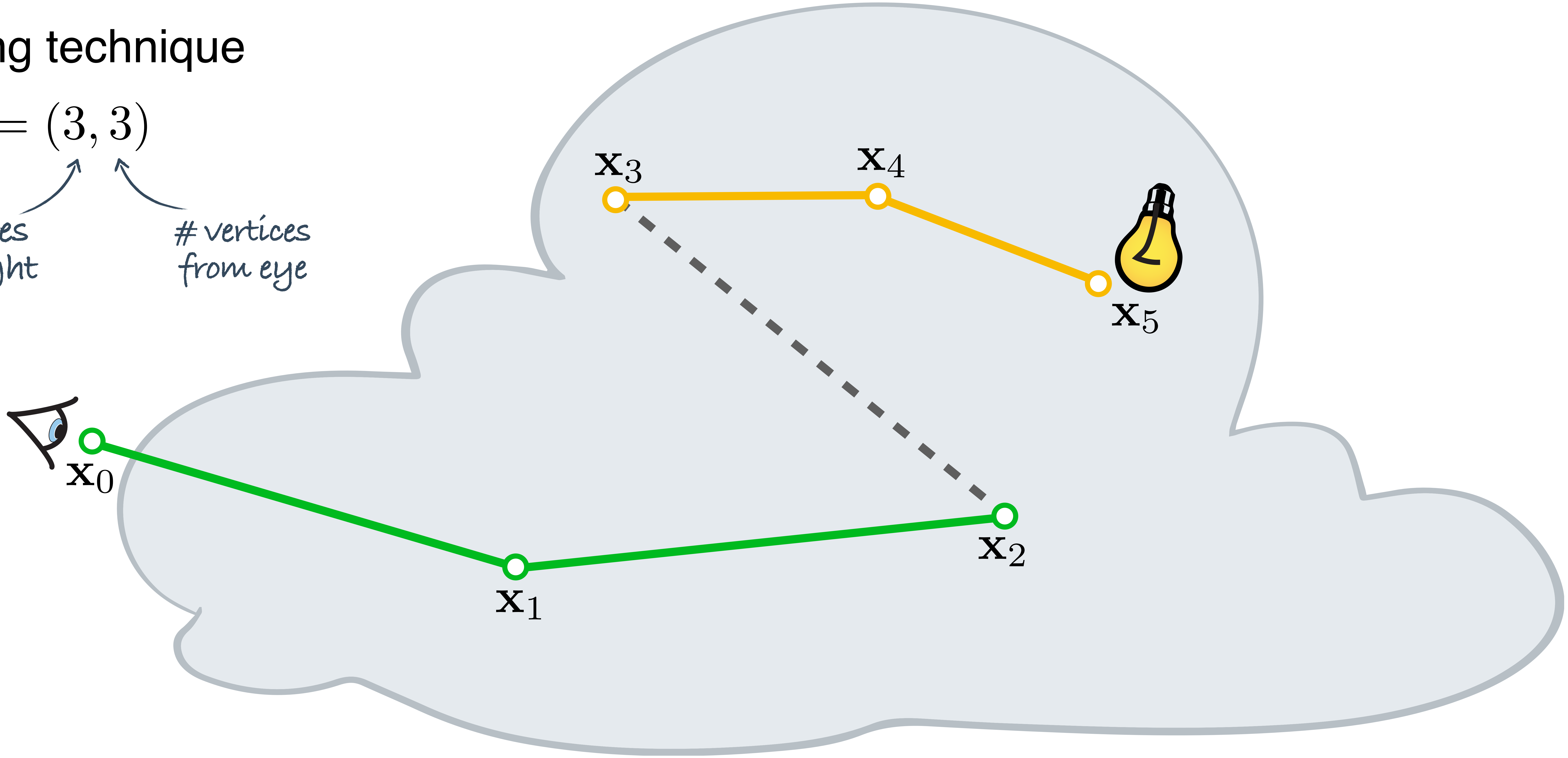
BIDIRECTIONAL PATH TRACING

Sampling technique

$$(s, t) = (3, 3)$$

vertices from light

vertices from eye



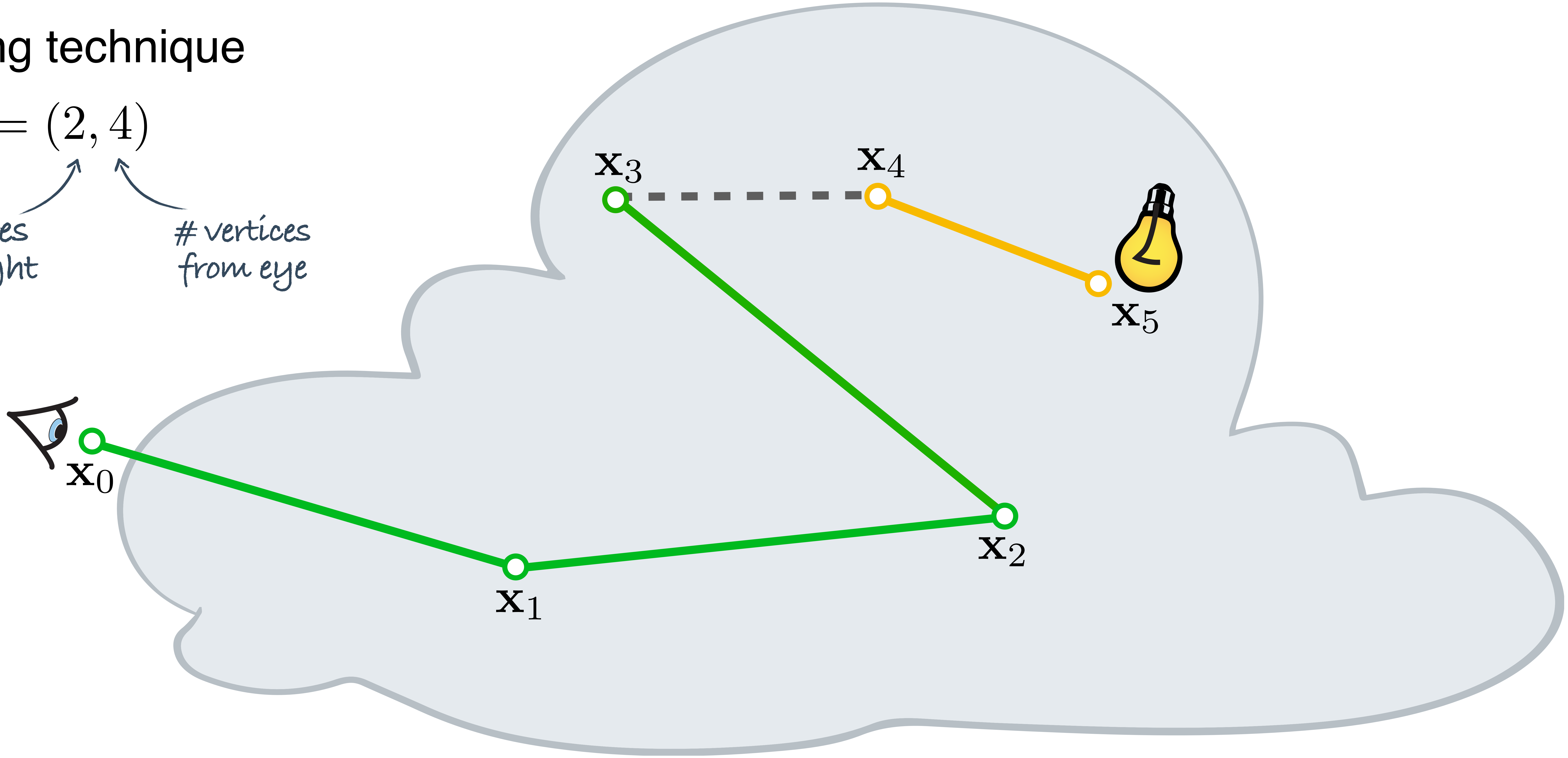
BIDIRECTIONAL PATH TRACING

Sampling technique

$$(s, t) = (2, 4)$$

vertices from light

vertices from eye



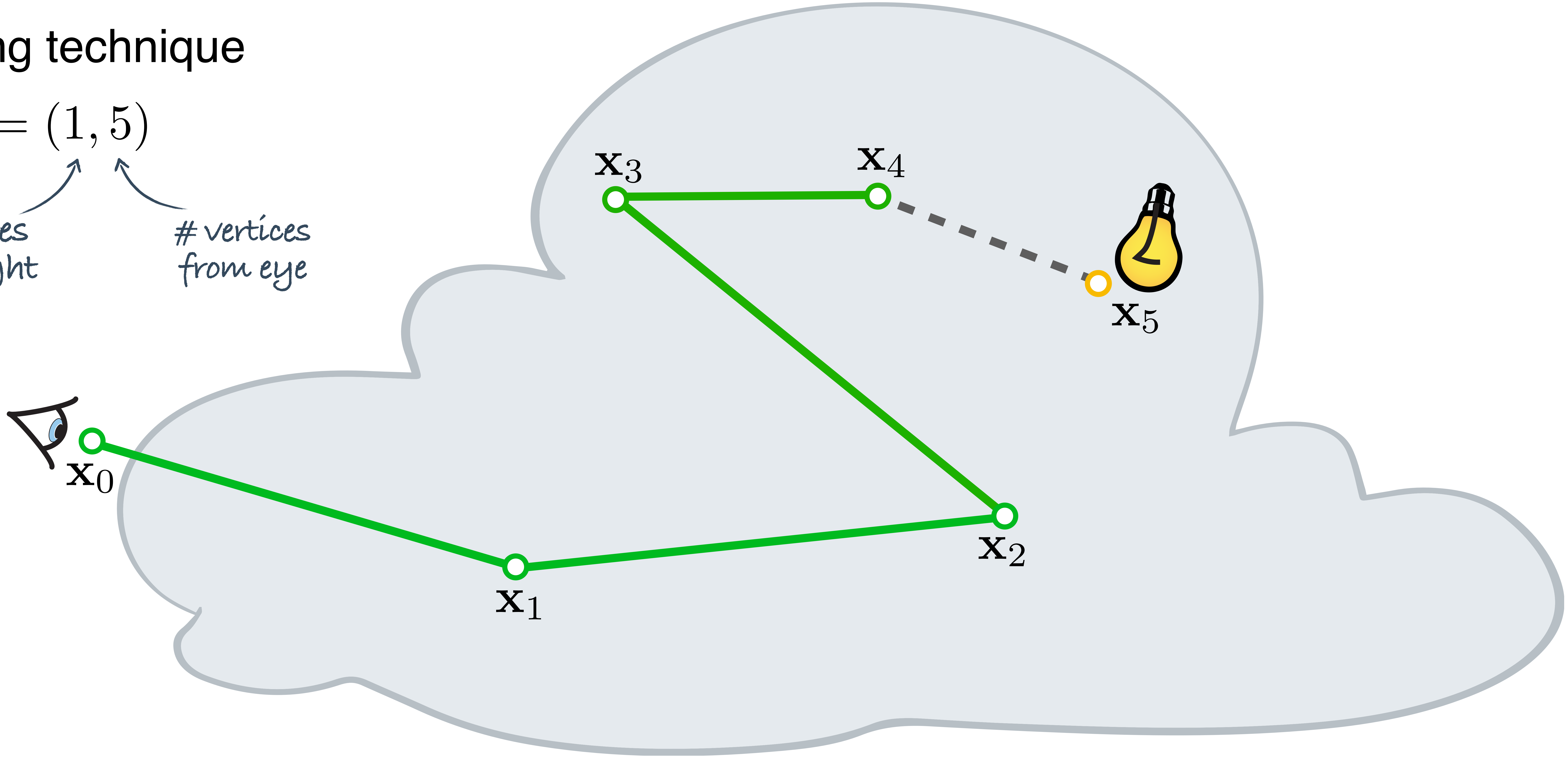
BIDIRECTIONAL PATH TRACING

Sampling technique

$$(s, t) = (1, 5)$$

vertices from light

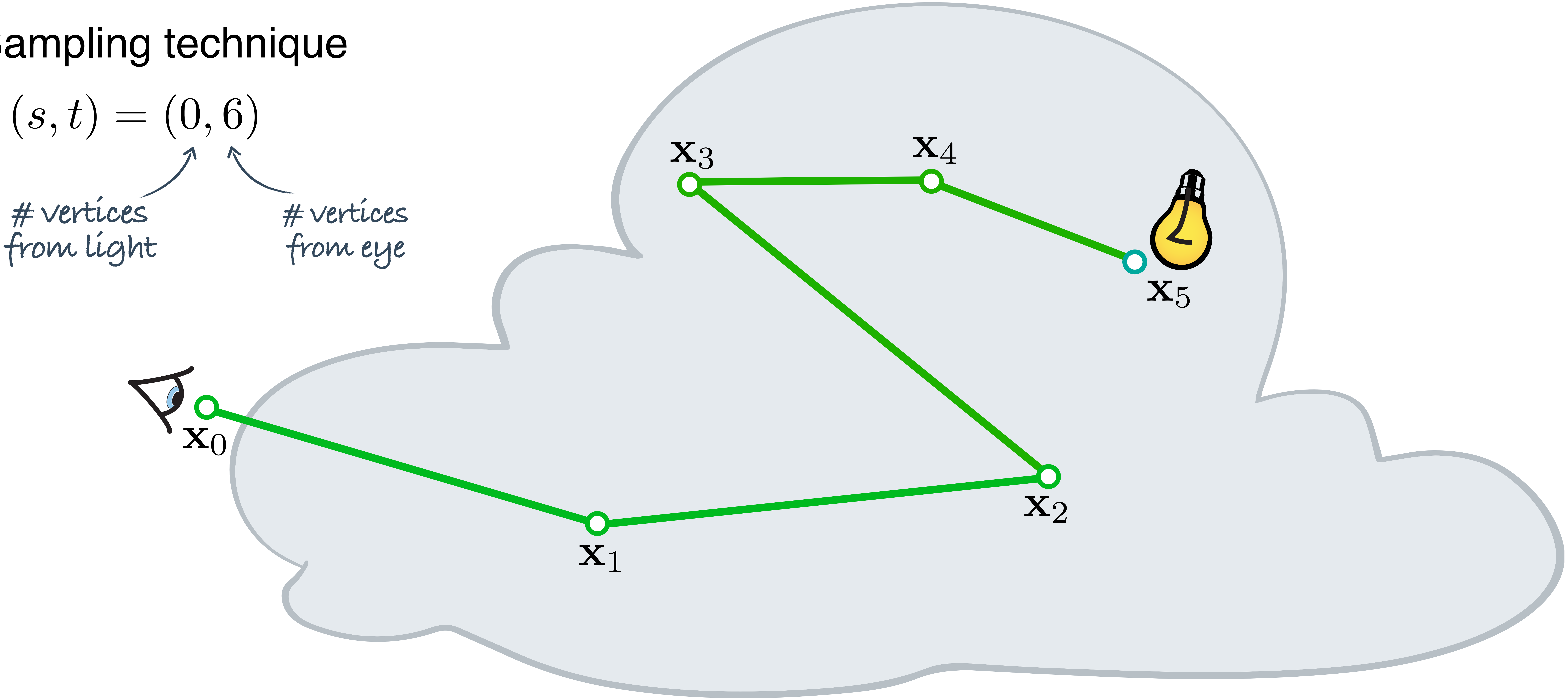
vertices from eye



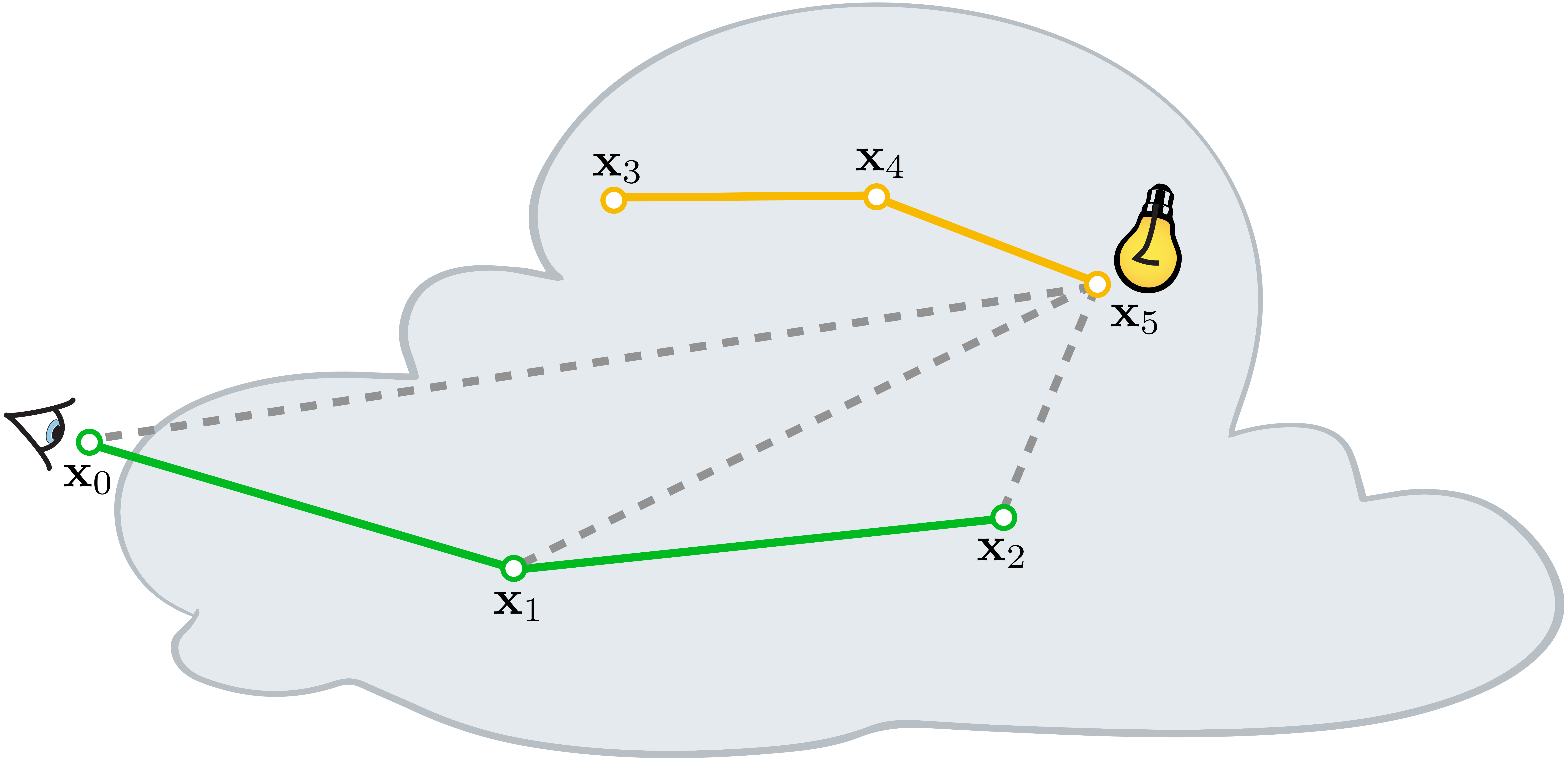
BIDIRECTIONAL PATH TRACING

Sampling technique

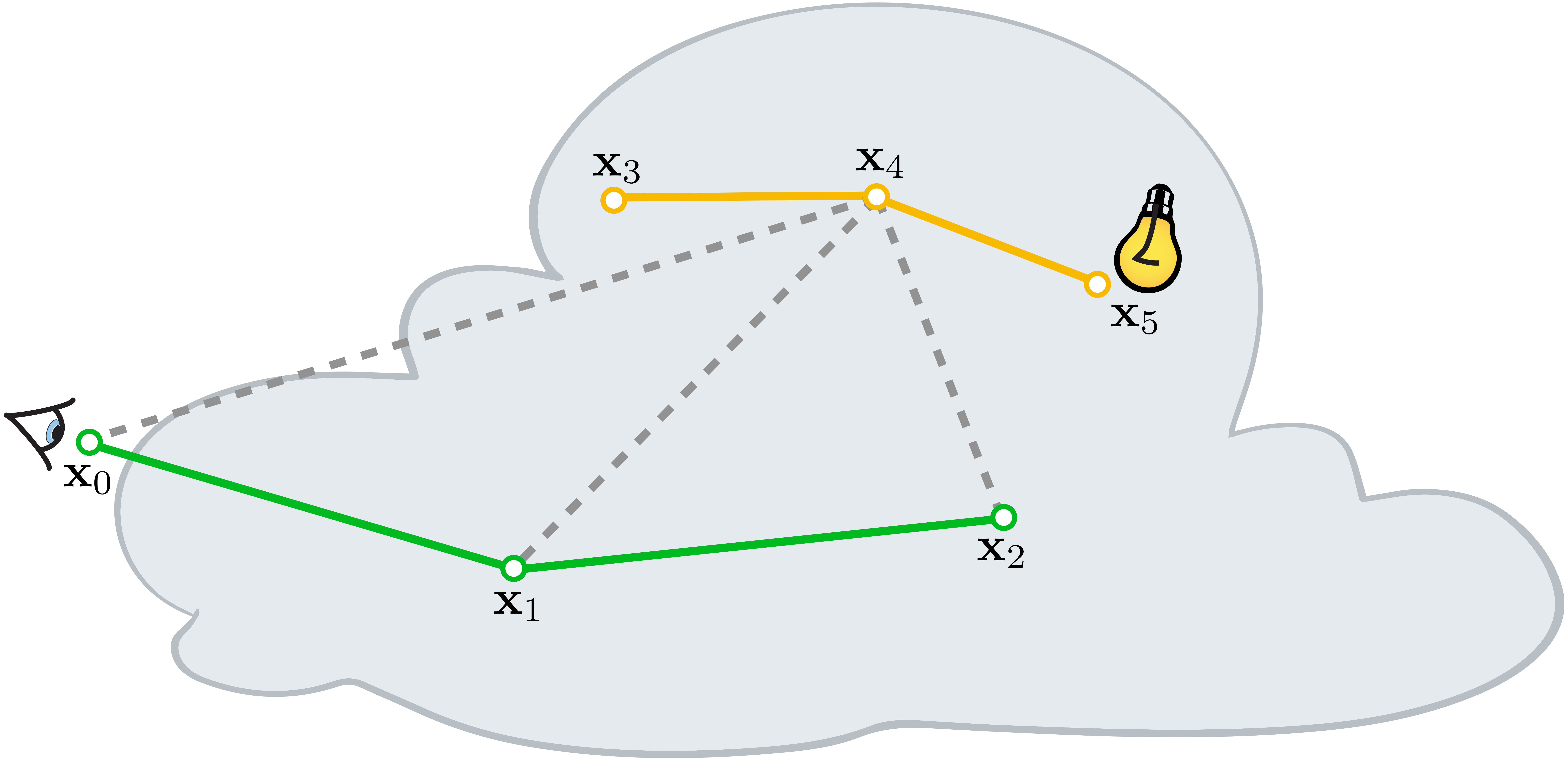
$$(s, t) = (0, 6)$$



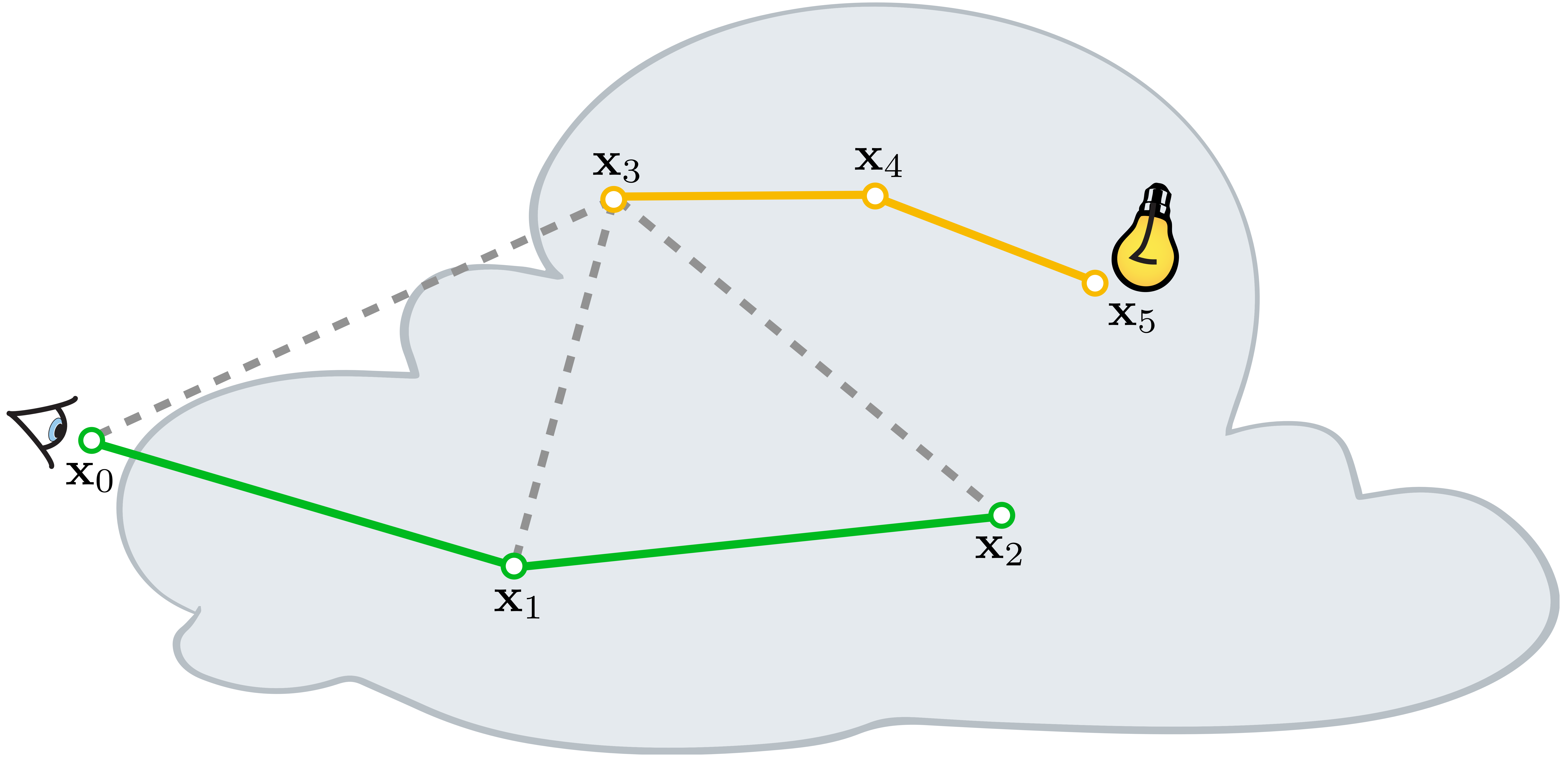
BIDIRECTIONAL PATH TRACING



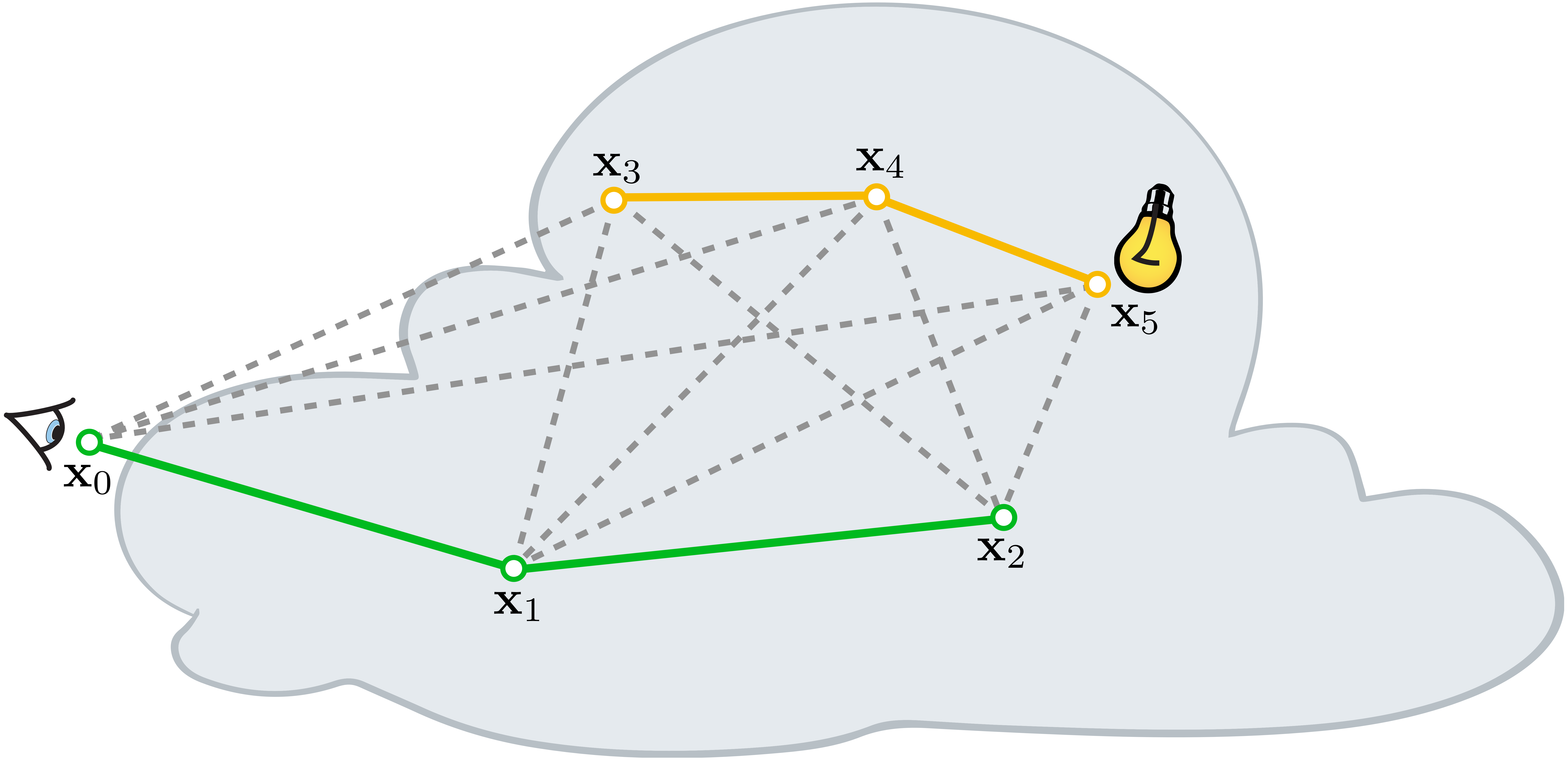
BIDIRECTIONAL PATH TRACING



BIDIRECTIONAL PATH TRACING



BIDIRECTIONAL PATH TRACING

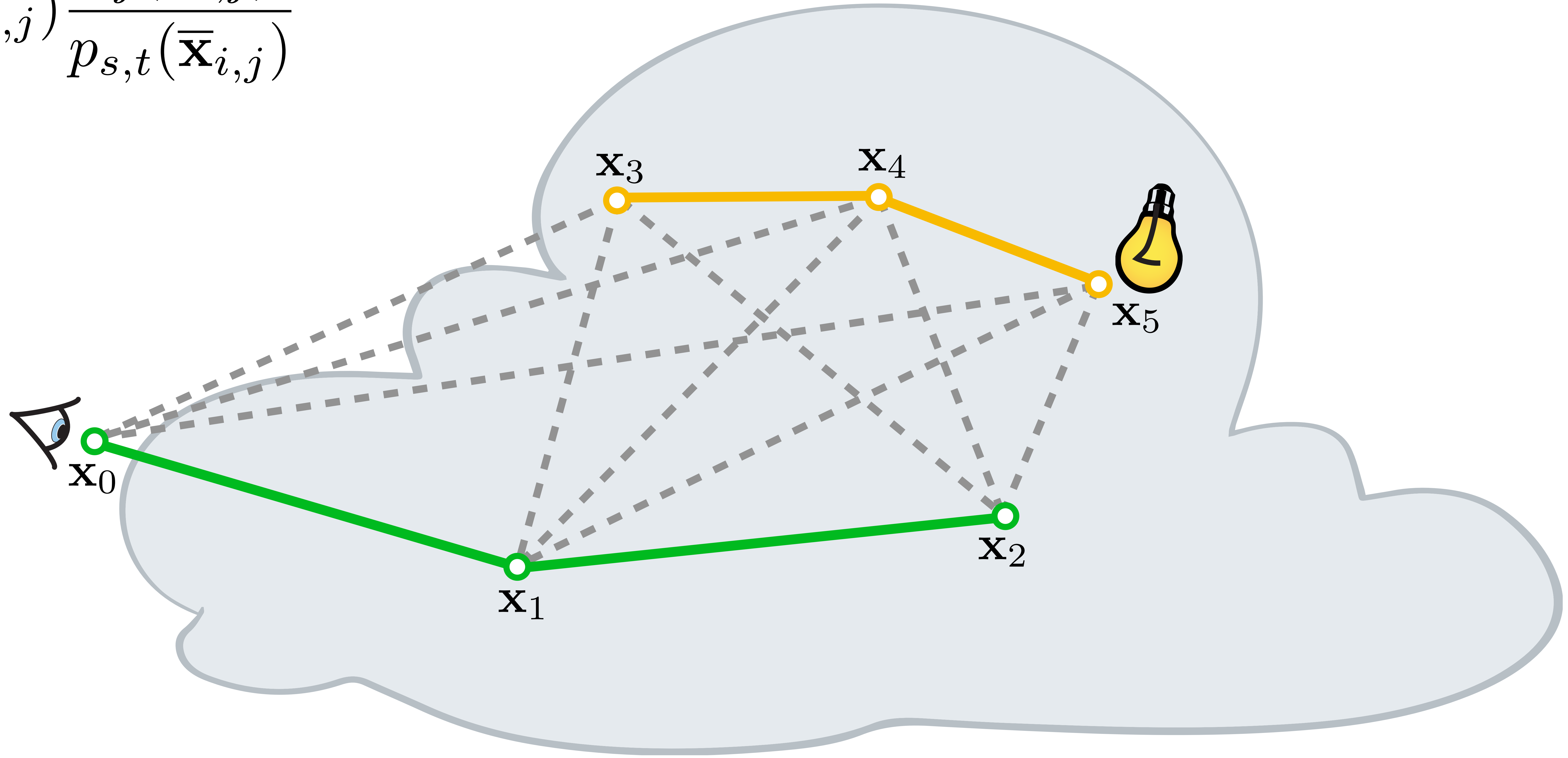


Combined MIS pixel estimator:

$$\langle I_j \rangle = \sum_{s \geq 0} \sum_{t \geq 0} w_{s,t}(\overline{\mathbf{x}}_{i,j}) \frac{f_j(\overline{\mathbf{x}}_{i,j})}{p_{s,t}(\overline{\mathbf{x}}_{i,j})}$$

vertices from light

vertices from eye



SUMMARY

UNIDIRECTIONAL SAMPLING

- Almost ideal on paper, rarely useful in practice

NEXT EVENT ESTIMATION

- Improvement, but singularity in indirect lighting (reduced convergence rate)

JOINT PATH SAMPLING

- Substantial improvement in the presence of singularities

BIDIRECTIONAL PATH TRACING

- Avoids singularities, more robust thanks to mixing many sampling techniques
- Difficult to implement